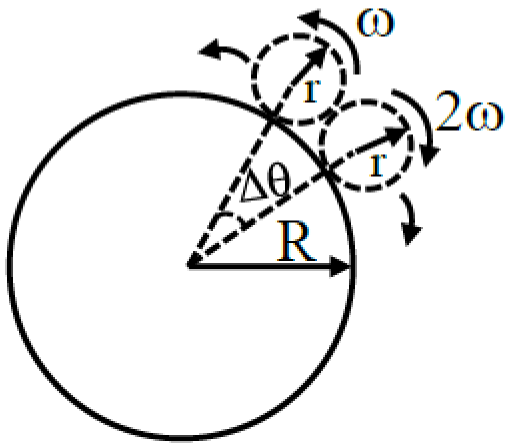
1. Consider a large disk of radius R and two smaller disks, each of radius r = R/50, lying on its circumference, as shown in the figure. The smaller disks are initially in contact with each other, with an angular separation Δθ between their centers. They are made to roll without slipping in opposite directions, with constant angular velocities ω and 2ω while the large disk is held stationary. The time ζ at which the smaller disks are again in contact is : [Use sin(Δθ) = Δθ and ignore gravity.]



(A) $\tau = 51\times\left(2\pi - \frac{4}{51}\right)/\omega$

(B) $\tau = 51\times\left(2\pi - \frac{2}{51}\right)/3\omega$

(C) $\tau = 51\times\left(2\pi - \frac{4}{51}\right)/3\omega$

(D) $\tau = 51\times\left(2\pi - \frac{2}{51}\right)/\omega$

$t = \frac{51}{3\omega}\left[2\pi - \frac{4}{51}\right]$

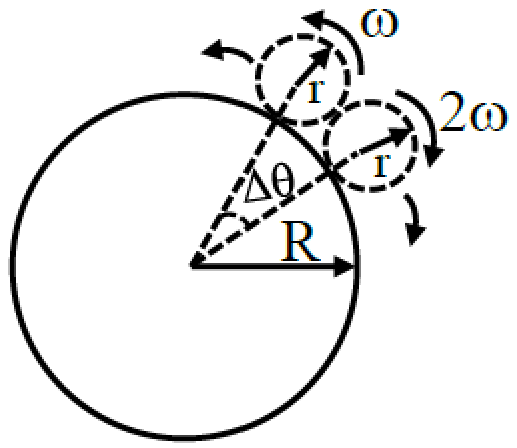
1. Consider a large disk of radius R and two smaller disks, each of radius r = R/50, lying on its circumference, as shown in the figure. The smaller disks are initially in contact with each other, with an angular separation Δθ between their centers. They are made to roll without slipping in opposite directions, with constant angular velocities ω and 2ω while the large disk is held stationary. The time ζ at which the smaller disks are again in contact is : [Use sin(Δθ) = Δθ and ignore gravity.]

(A) $\tau = 51\times\left(2\pi - \frac{4}{51}\right)/\omega$

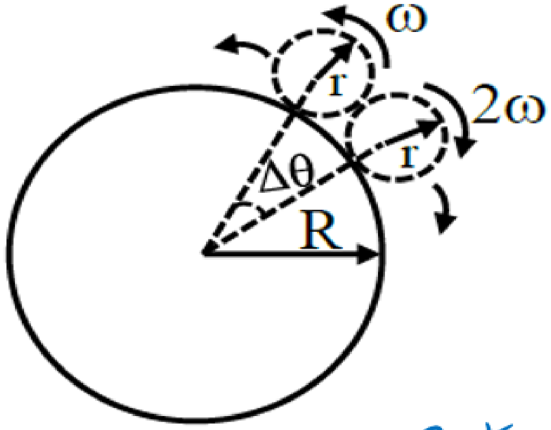
(B) $\tau = 51\times\left(2\pi - \frac{2}{51}\right)/3\omega$

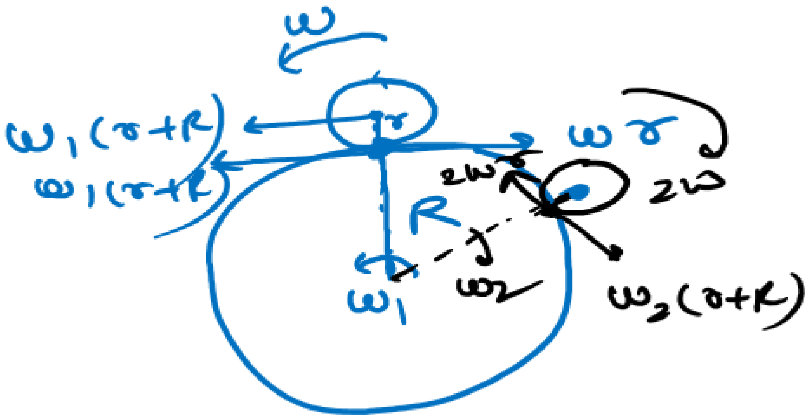
(C) $\tau = 51\times\left(2\pi - \frac{4}{51}\right)/3\omega$

(D) $\tau = 51\times\left(2\pi - \frac{2}{51}\right)/\omega$



1. Consider a large disk of radius R and two smaller disks, each of radius r = R/50, lying on its circumference, as shown in the figure. The smaller disks are initially in contact with each other, with an angular separation Δθ between their centers. They are made to roll without slipping in opposite directions, with constant angular velocities ω and 2ω while the large disk is held stationary. The time ζ at which the smaller disks are again in contact is : [Use sin(Δθ) = Δθ and ignore gravity.]





$$r = R/50$$

$$\omega_1(r+R) = \omega r$$

$$\omega_1(r+50r) = \omega r$$

$$\omega_1 = \frac{\omega}{51}$$

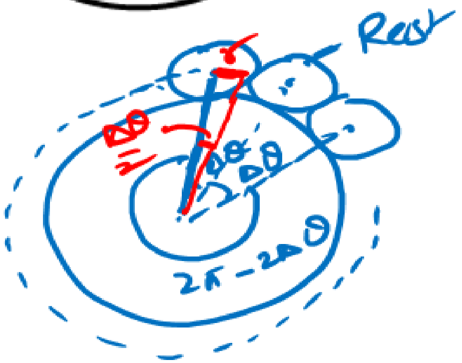
$$\omega_2(r+R) = 2\omega r$$

$$\omega_2(r+50r) = 2\omega r$$

$$\omega_2 = \frac{2\omega}{51}$$

$$\frac{\Delta\theta}{2} = \frac{r}{R+r} = \frac{r}{50r+r} = \frac{1}{51}$$

$$\Delta\theta = \frac{2}{51}$$

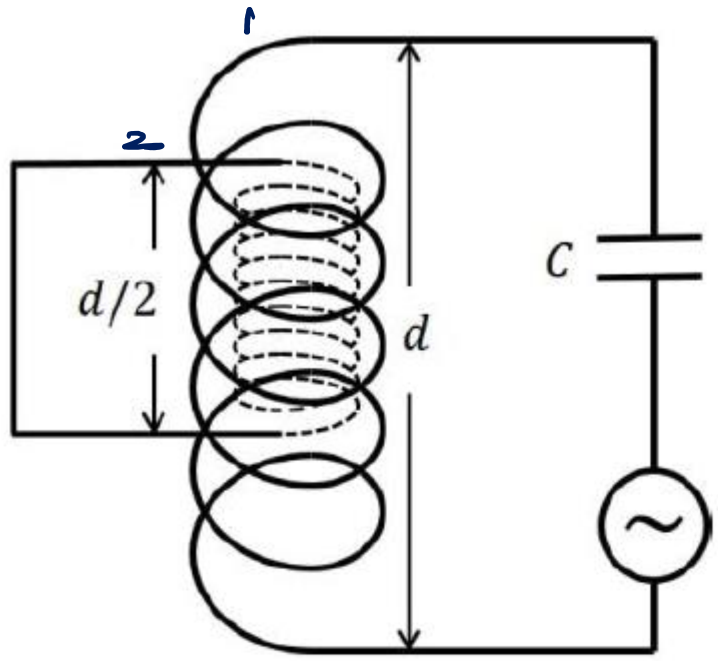


$$\omega_1+\omega_2 = \frac{2\pi-2\Delta\theta}{t}$$

$$t = \frac{2\pi-2\Delta\theta}{\omega_1+\omega_2} = \frac{2\pi-\frac{4}{51}}{3\omega/51}$$

$$t = \frac{51}{3\omega}\left[2\pi-\frac{4}{51}\right]$$

2. Consider a circuit consisting of a capacitor of capacitance C and a coil with N turns per unit length, cross sectional area S and length d, where $d^2 \gg S$. There is another coil of length $d/2$, cross sectional area $S/2$ and $2N$ turns per unit length completely inside the larger coil, as shown in the figure. The ends of this smaller coil are connected with each other by an insulated conducting wire. The self inductance of the larger coil is L. Neglecting edge effects and all the Ohmic resistances, the resonant frequency of the circuit is:



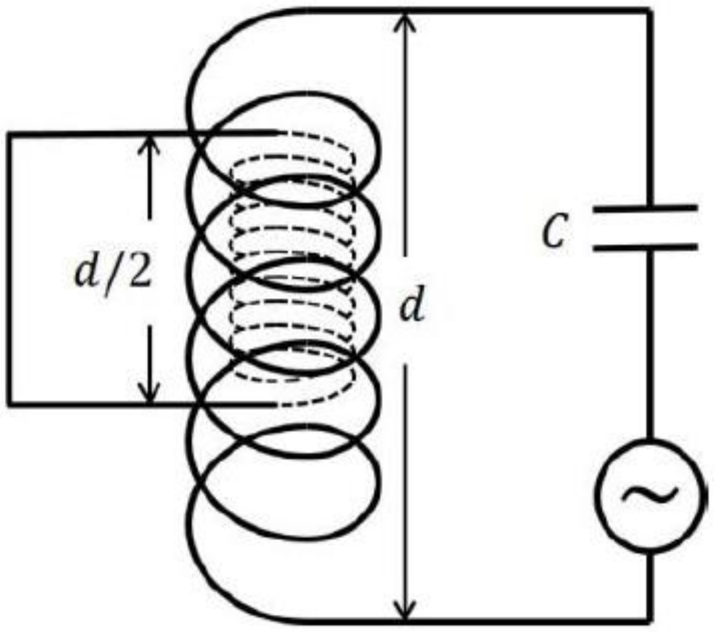
(A) $\dfrac{4}{\sqrt{15LC}}$

(B) $\dfrac{6}{\sqrt{5LC}}$

(C) $\dfrac{2}{\sqrt{3LC}}$

(D) $\sqrt{\dfrac{2}{3LC}}$

2. Consider a circuit consisting of a capacitor of capacitance C and a coil with N turns per unit length, cross sectional area S and length d, where $d^2 \gg S$. There is another coil of length $d/2$, cross sectional area $S/2$ and $2N$ turns per unit length completely inside the larger coil, as shown in the figure. The ends of this smaller coil are connected with each other by an insulated conducting wire. The self inductance of the larger coil is L. Neglecting edge effects and all the Ohmic resistances, the resonant frequency of the circuit is:



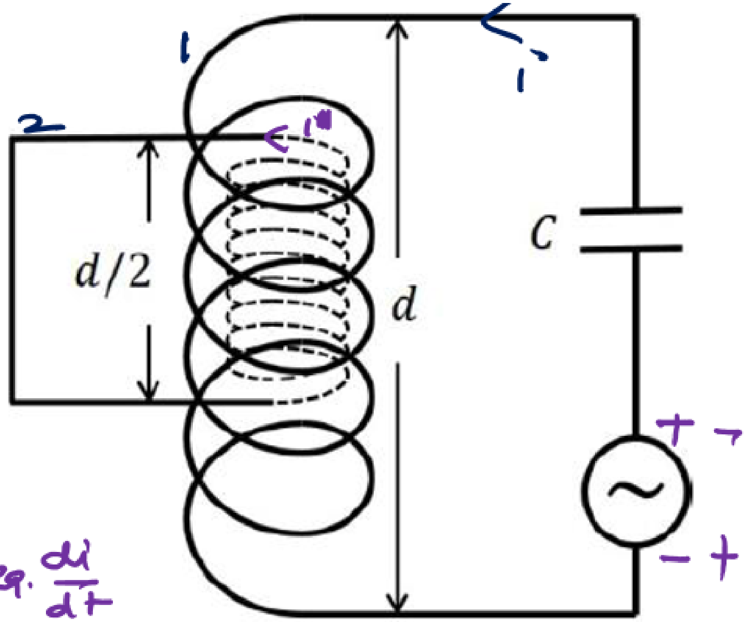
(A) $\frac{4}{\sqrt{15LC}}$

(B) $\frac{6}{\sqrt{5LC}}$

(C) $\frac{2}{\sqrt{3LC}}$

(D) $\sqrt{\frac{2}{3LC}}$

2. Consider a circuit consisting of a capacitor of capacitance C and a coil with N turns per unit length, cross sectional area S and length d, where $d^2 \gg S$. There is another coil of length $d/2$, cross sectional area $S/2$ and $2N$ turns per unit length completely inside the larger coil, as shown in the figure. The ends of this smaller coil are connected with each other by an insulated conducting wire. The self inductance of the larger coil is L. Neglecting edge effects and all the Ohmic resistances, the resonant frequency of the circuit is:



1 → $n_1 = N$, $A_1 = S$, $\ell_1 = d$

2 → $n_2 = 2N$, $A_2 = S/2$, $\ell_2 = \frac{d}{2}$

$$f = \frac{1}{\sqrt{L_{eq}C}} = \frac{1}{\sqrt{\frac{3L}{4}C}}$$

Calculate self inductance → 1

$$\phi = B.A = (\mu_0 n_1 i) S N d$$

$$\phi = \mu_0 S N^2 d i = L i$$

$$L = \mu_0 S N^2 d$$

$$V_1 = L\frac{di}{dt} - \frac{M^2}{L}\frac{di}{dt}$$

$$V_1 = L\frac{di}{dt} - \frac{L^2}{4\cancel{L}}\frac{di}{dt}$$

$$V_1 = \frac{di}{dt}\left(\frac{3L}{4}\right) = L_{eq}\frac{di}{dt}$$

Similarly self inductance → 2

$$L_2 = \mu_0 \frac{S}{2}(2N)^2\frac{d}{2} = \mu_0 S\frac{\cancel{4}N^2 d}{\cancel{4}} = L$$

Pot. diff across 1 → $V_1 = L\frac{di}{dt} - M\frac{di'}{dt}$

Pot. diff across 2 → $V_2 = L\frac{di'}{dt} - M\frac{di}{dt} = 0 \Rightarrow \frac{di'}{dt} = \frac{M}{L}\frac{di}{dt}$

Mutual Inductance

$$\phi_2 = \mu_0 N i \frac{S}{\cancel{2}} \cancel{2} N \frac{d}{2} = \frac{\mu_0 N^2 S d i}{2}$$

$$\phi_2 = M i$$

$$M = \frac{\mu_0 N^2 S d}{2} = \frac{L}{2}$$

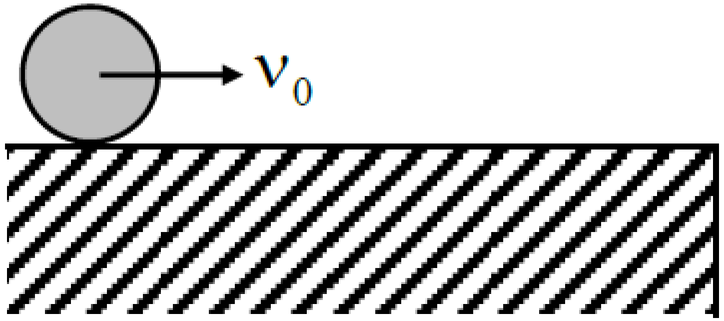
3. A solid cylinder of radius R rolls without slipping with a center of mass speed $V_0 = \sqrt{(gR/3)}$ on a horizontal surface with a vertical edge, as shown in the figure. Here, g is the acceleration due to the gravity. At the moment when the cylinder loses contact with the surface due to rotation around the corner, the speed of its center of mass is:

(A) 0

(B) $\sqrt{\frac{5gR}{7}}$

(C) $\sqrt{\frac{gR}{15}}$

(D) $\sqrt{\frac{3gR}{7}}$



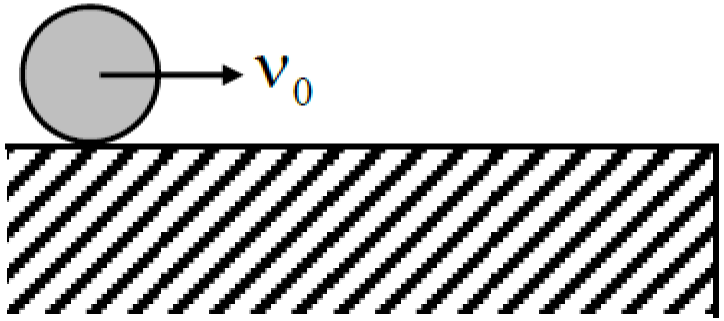
3. A solid cylinder of radius R rolls without slipping with a center of mass speed $V_0 = \sqrt{(gR/3)}$ on a horizontal surface with a vertical edge, as shown in the figure. Here, g is the acceleration due to the gravity. At the moment when the cylinder loses contact with the surface due to rotation around the corner, the speed of its center of mass is:

(A) 0

(B) $\sqrt{\frac{5gR}{7}}$

(C) $\sqrt{\frac{gR}{15}}$

(D) $\sqrt{\frac{3gR}{7}}$



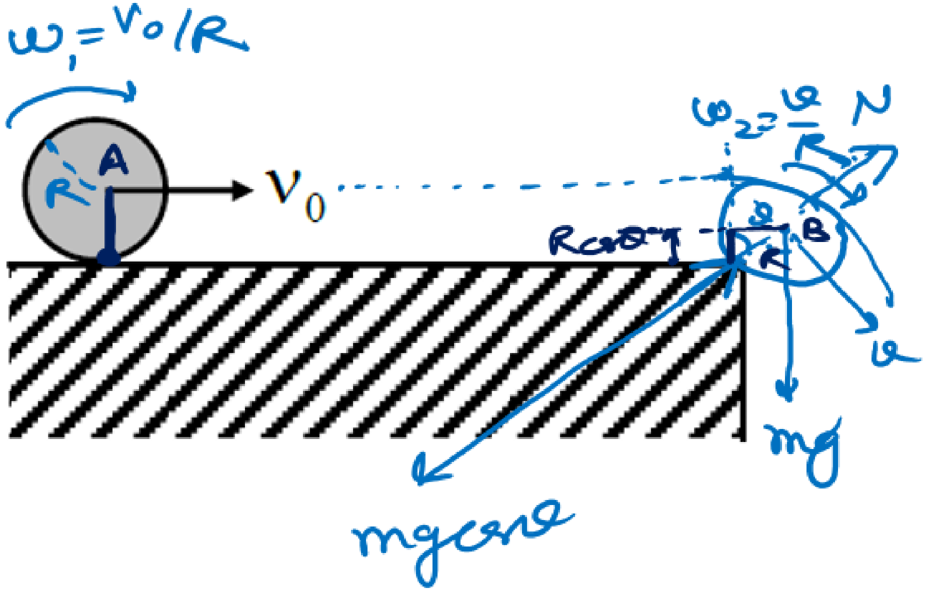
3. A solid cylinder of radius R rolls without slipping with a center of mass speed $V_0 = \sqrt{(gR/3)}$ on a horizontal surface with a vertical edge, as shown in the figure. Here, g is the acceleration due to the gravity. At the moment when the cylinder loses contact with the surface due to rotation around the corner, the speed of its center of mass is:

(A) 0

(B) $\sqrt{\frac{5gR}{7}}$

(C) $\sqrt{\frac{gR}{15}}$

(D) $\sqrt{\frac{3gR}{7}}$



$T.E_A = T.E_B$

$mgR + \frac{1}{2} I \omega_1^2 = mgR\cos\theta + \frac{1}{2} I \omega_2^2$

$mgR + \frac{1}{2}\cdot\frac{3}{2} MR^2 \frac{v_0^2}{R^2} = mgR\cos\theta + \frac{1}{2}\cdot\frac{3}{2} MR^2 \frac{v^2}{R^2}$

$gR + \frac{3}{4}\frac{gR}{3} = gR\cos\theta + \frac{3}{4}v^2 \Rightarrow \frac{5}{4} gR = \frac{7}{4} v^2 \Rightarrow v^2 = \frac{5}{7} gR$

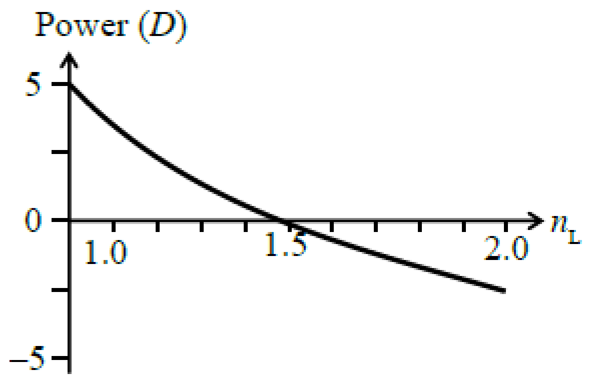
$mg\cos\theta - N = \frac{mv^2}{R}$

$N = 0$

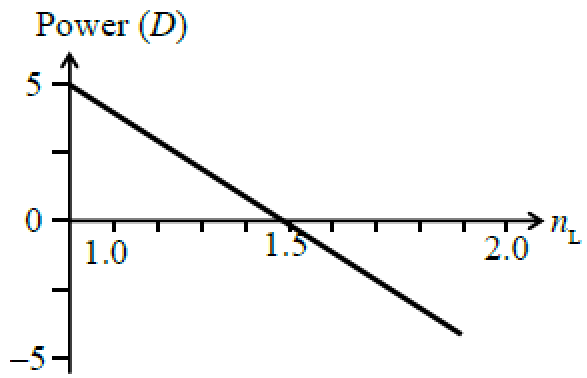
$mg\cos\theta = \frac{mv^2}{R} \Rightarrow v^2 = gR\cos\theta$

4. A double convex lens made of glass of refractive index 1.5 and radii of curvature of the curved surfaces 20 cm each is immersed in a liquid of refractive index n_L. The correct plot showing the variation of the power, in the units of diopter (D), as a function of n_L is

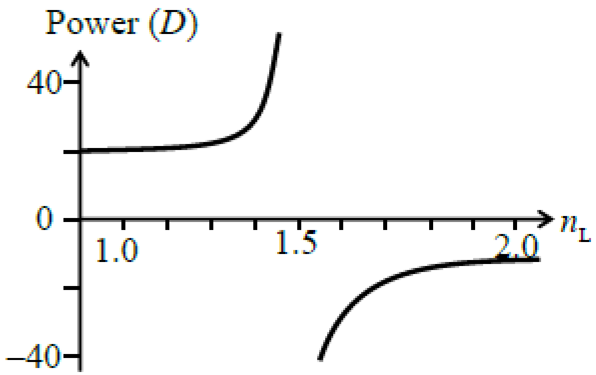
(A)



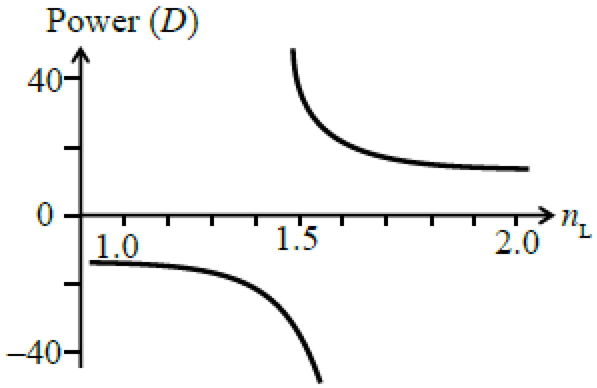
(B)



(C)

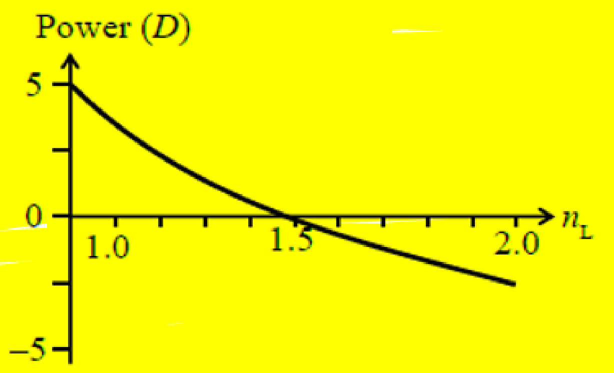


(D)

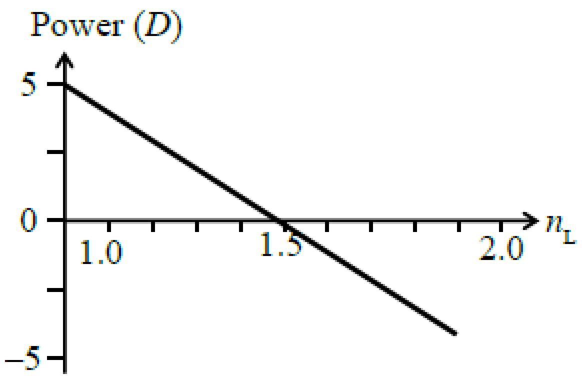


4. A double convex lens made of glass of refractive index 1.5 and radii of curvature of the curved surfaces 20 cm each is immersed in a liquid of refractive index n_L. The correct plot showing the variation of the power, in the units of diopter (D), as a function of n_L is

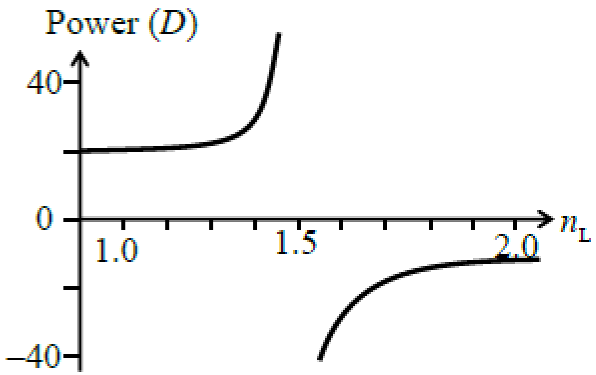
(A)



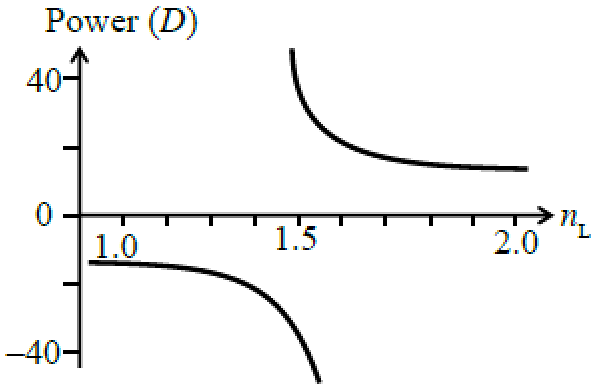
(B)



(C)

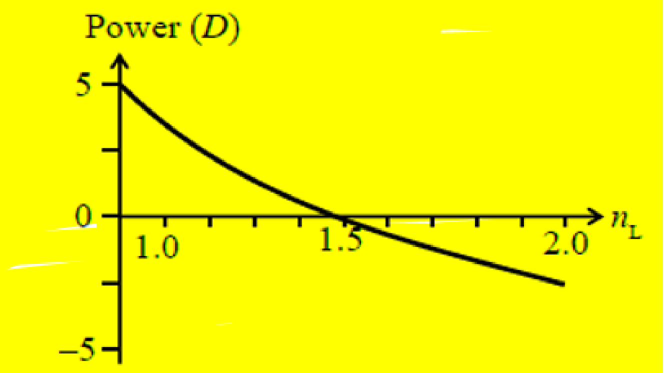


(D)

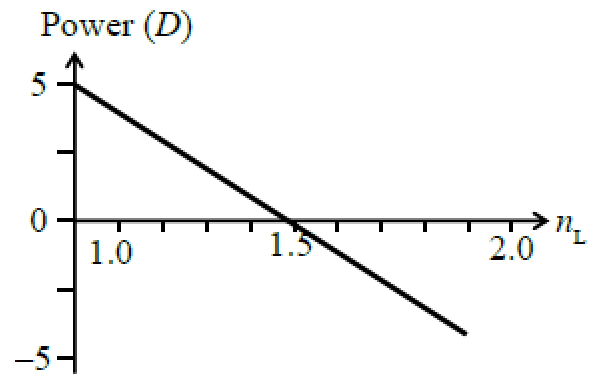


4. A double convex lens made of glass of refractive index 1.5 and radii of curvature of the curved surfaces 20 cm each is immersed in a liquid of refractive index n_L. The correct plot showing the variation of the power, in the units of diopter (D), as a function of n_L is

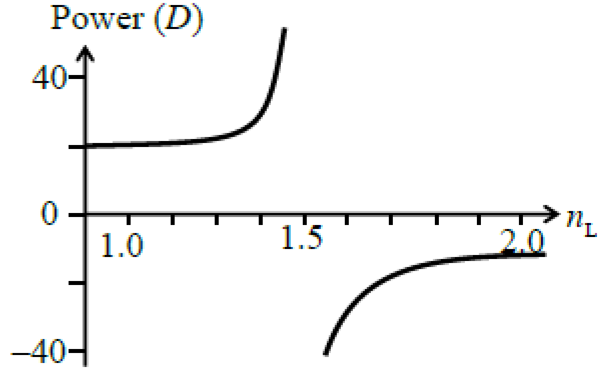
(A)



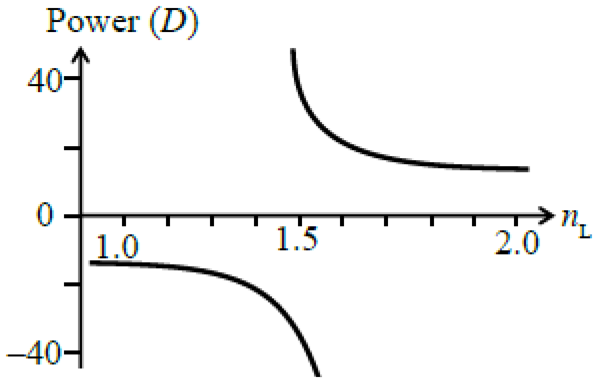
(B)



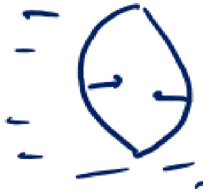
(C)



(D)



$\mu_g = \frac{3}{2}$



$R_1 = 20\ cm = 0.2\,m$

$R_2 = -20\,cm = -0.2\,m$

$$P = \frac{1}{f} = \left(\frac{\mu_g}{\mu_s} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$P = \left(\frac{3}{2n_L} - 1\right)\left(\frac{1}{0.2} + \frac{1}{0.2}\right)$$

$$P = \left(\frac{3}{2n_L} - 1\right)\left(\frac{2}{0.2}\right)$$

$$P = \left(\frac{3}{2n_L} - 1\right)(10)$$

$n_L = 1$, $P = 0.5 \times 10 = 5$

$n_L = 2$, $P = -\frac{1}{4} \times 10 = -2.5$

Q.5 Consider a hydrogen atom with v_k, r_k, and K_k denoting the velocity, orbital radius and kinetic energy of the electron in the k^{th} orbit, respectively. The electron undergoes a transition from the n^{th} orbit, emitting radiation corresponding to the Lyman series. Considering h to be the Planck's constant and ϵ_0 the permittivity of the free space, the correct statement(s) is/are:

(A)	Magnitude of change in kinetic energy of electron can be expressed as $\frac{h}{4\pi}\left\|\frac{nv_n}{r_n} - \frac{v_1}{r_1}\right\|$.
(B)	Magnitude of change in de Broglie wavelength of the electron can be expressed as $\frac{e^2}{4\epsilon_0}\left\|\frac{1}{K_n} - \frac{1}{K_1}\right\|$.
(C)	Frequency of the radiation emitted can be expressed as $\frac{e^2}{8\pi\epsilon_0 h}\left(\frac{1}{r_1} - \frac{1}{r_n}\right)$.
(D)	Magnitude of change in total energy of the electron can be expressed as $\frac{h}{2\pi}\left\|\frac{v_1}{r_1} - \frac{nv_n}{r_n}\right\|$.

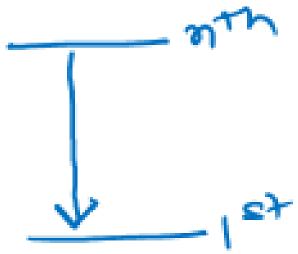
Q.5 Consider a hydrogen atom with v_k, r_k, and K_k denoting the velocity, orbital radius and kinetic energy of the electron in the k^{th} orbit, respectively. The electron undergoes a transition from the n^{th} orbit, emitting radiation corresponding to the Lyman series. Considering h to be the Planck's constant and ϵ_0 the permittivity of the free space, the correct statement(s) is/are:

(A)	Magnitude of change in kinetic energy of electron can be expressed as $\frac{h}{4\pi}\left\lvert\frac{nv_n}{r_n}-\frac{v_1}{r_1}\right\rvert$.
(B)	Magnitude of change in de Broglie wavelength of the electron can be expressed as $\frac{e^2}{4\epsilon_0}\left\lvert\frac{1}{K_n}-\frac{1}{K_1}\right\rvert$.
(C)	Frequency of the radiation emitted can be expressed as $\frac{e^2}{8\pi\epsilon_0 h}\left(\frac{1}{r_1}-\frac{1}{r_n}\right)$.
(D)	Magnitude of change in total energy of the electron can be expressed as $\frac{h}{2\pi}\left\lvert\frac{v_1}{r_1}-\frac{nv_n}{r_n}\right\rvert$.

A & C

Q.5 Consider a hydrogen atom with v_k, r_k, and K_k denoting the velocity, orbital radius and kinetic energy of the electron in the k^{th} orbit, respectively. The electron undergoes a transition from the n^{th} orbit, emitting radiation corresponding to the Lyman series. Considering h to be the Planck's constant and ϵ_0 the permittivity of the free space, the correct statement(s) is/are:

(A)	Magnitude of change in kinetic energy of electron can be expressed as $\frac{h}{4\pi}\left\|\frac{nv_n}{r_n} - \frac{v_1}{r_1}\right\|$.



$$KE = \frac{1}{2} m v_n^2$$

$$= \frac{1}{2} m v_n v_n$$

$$m v_n r_n = \frac{nh}{2\pi}$$

$$m v_n = \frac{nh}{2\pi r_n}$$

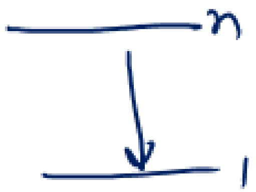
$$KE_n = \frac{1}{2} \cdot \frac{nh}{2\pi r_n} v_n = \frac{nh v_n}{4\pi r_n}$$

$$KE_1 = \frac{1}{2} \frac{h \cdot v}{2\pi r_1} = \frac{h v_1}{4\pi r_1}$$

$$\Delta K = KE_n - KE_1 = \frac{h}{4\pi}\left[\frac{n v_n}{r_n} - \frac{v_1}{r_1}\right]$$

Q.5 Consider a hydrogen atom with v_k, r_k, and K_k denoting the velocity, orbital radius and kinetic energy of the electron in the k^{th} orbit, respectively. The electron undergoes a transition from the n^{th} orbit, emitting radiation corresponding to the Lyman series. Considering h to be the Planck's constant and ϵ_0 the permittivity of the free space, the correct statement(s) is/are:

(B)	Magnitude of change in de Broglie wavelength of the electron can be expressed as $\frac{e^2}{4\epsilon_0}\left\lvert\frac{1}{K_n} - \frac{1}{K_1}\right\rvert$.



$\lambda_n = \frac{h\, r_n}{m v_n r_n}$

$\lambda_n = \frac{\cancel{h}\, r_n\, 2\pi}{n \cancel{h}}$

$\lambda_n = \frac{2\pi r_n}{n}$

$\lambda_n = \frac{\cancel{2\pi}}{n} \frac{e^2}{\cancel{8}4\cancel{\pi}\epsilon_0 K_n} = \frac{e^2}{4n\epsilon_0 K_n}$

$\lambda_1 = \frac{e^2}{4\epsilon_0 K_1}$

$\frac{K q_1 q_2}{r^2} = \frac{m v^2}{r}$

$\frac{1}{2} \frac{K \cdot e^2}{\cancel{r_n^2}} = \frac{1}{2} \frac{m v_n^2}{\cancel{r_n}}$

$K_n = \frac{1}{2} \frac{K e^2}{r_n} = \frac{e^2}{8\pi\epsilon_0 r_n}$

$r_n = \frac{e^2}{8\pi\epsilon_0 K_n}$

$\Delta\lambda = \frac{e^2}{4\epsilon_0}\left[\frac{1}{nK_n} - \frac{1}{K_1}\right]$

Q.5 Consider a hydrogen atom with v_k, r_k, and K_k denoting the velocity, orbital radius and kinetic energy of the electron in the k^{th} orbit, respectively. The electron undergoes a transition from the n^{th} orbit, emitting radiation corresponding to the Lyman series. Considering h to be the Planck's constant and ϵ_0 the permittivity of the free space, the correct statement(s) is/are:

(C)	Frequency of the radiation emitted can be expressed as $\frac{e^2}{8\pi\epsilon_0 h}\left(\frac{1}{r_1} - \frac{1}{r_n}\right)$.

$$E = K + P = \frac{1}{2}mv^2 + \left(-\frac{kZe^2}{r}\right)$$

$$\frac{1}{2}\frac{mv^2}{r} = \frac{1}{2}\frac{kZe^2}{r^2}$$

$$= \frac{kZe^2}{2r} - \frac{kZe^2}{r} = -\frac{kZe^2}{2r}$$

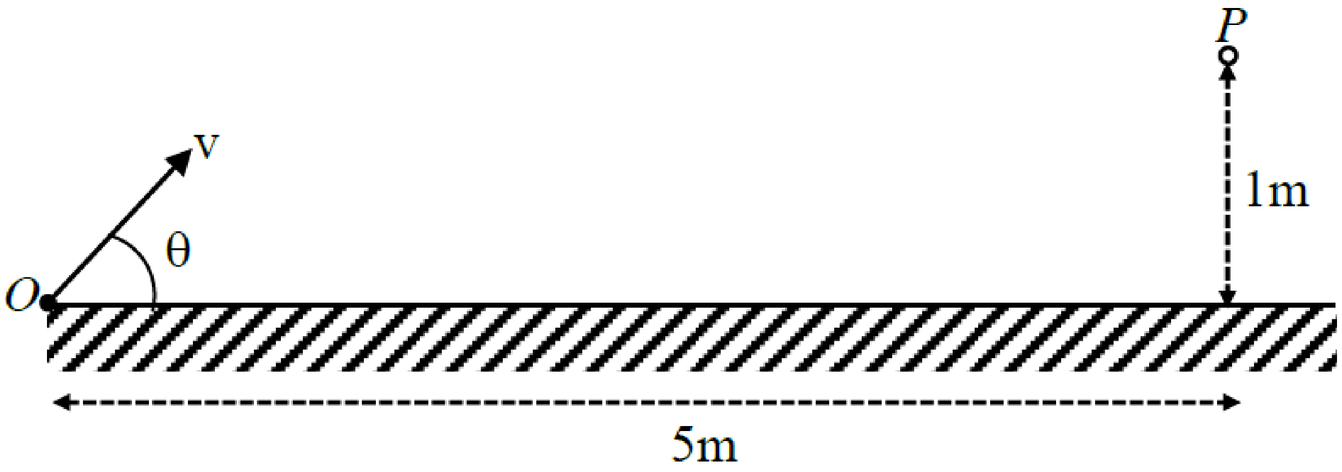
$$\frac{kq_1q_2}{r} = -\frac{kZee}{r}$$

$$E_n = -\frac{ke^2}{2r} = -\frac{e^2}{8\pi\epsilon_0 r_n}$$

$$\Delta E = E_n - E_1 = h\nu$$

$$\nu = \frac{-e^2}{8\pi\epsilon_0 r_n h} + \frac{e^2}{8\pi\epsilon_0 r_1 h} = \frac{e^2}{8\pi\epsilon_0 h}\left[\frac{1}{r_1} - \frac{1}{r_n}\right)$$

6. A particle is thrown with a speed v from a point O at an angle θ with the horizontal plane such that it passes through the point P at a height of 1 m and horizontal distance of 5 m from O, as shown in the figure. If acceleration due to gravity is g ms^{-2}, then the correct statement (s) is/are :



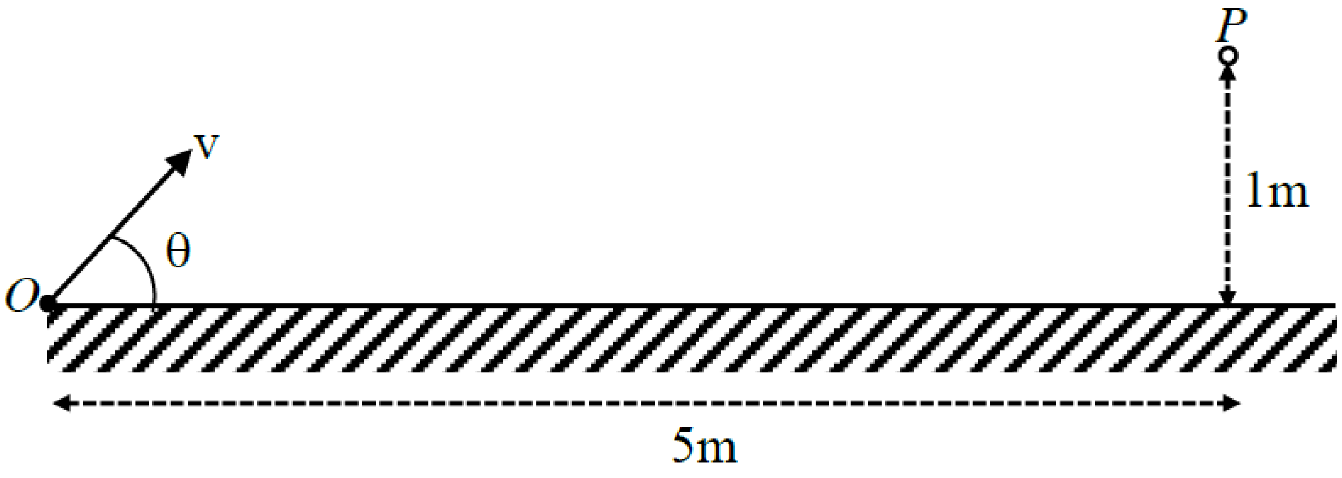
(A) If $\theta = 45°$, then $v = \dfrac{5\sqrt{g}}{2} ms^{-1}$

(B) If $\theta = 45°$, the particle reaches its maximum height before it reaches P.

(C) If $\theta = 30°$, the particle reaches its maximum height after reaching P.

(D) If $\theta = \tan^{-1}\left(\dfrac{1}{5}\right)$, then $v = 125\sqrt{g}\ ms^{-1}$

6. A particle is thrown with a speed v from a point O at an angle θ with the horizontal plane such that it passes through the point P at a height of 1 m and horizontal distance of 5 m from O, as shown in the figure. If acceleration due to gravity is g ms^{-2}, then the correct statement (s) is/are :



(A) If $\theta = 45°$, then $v = \frac{5\sqrt{g}}{2} ms^{-1}$

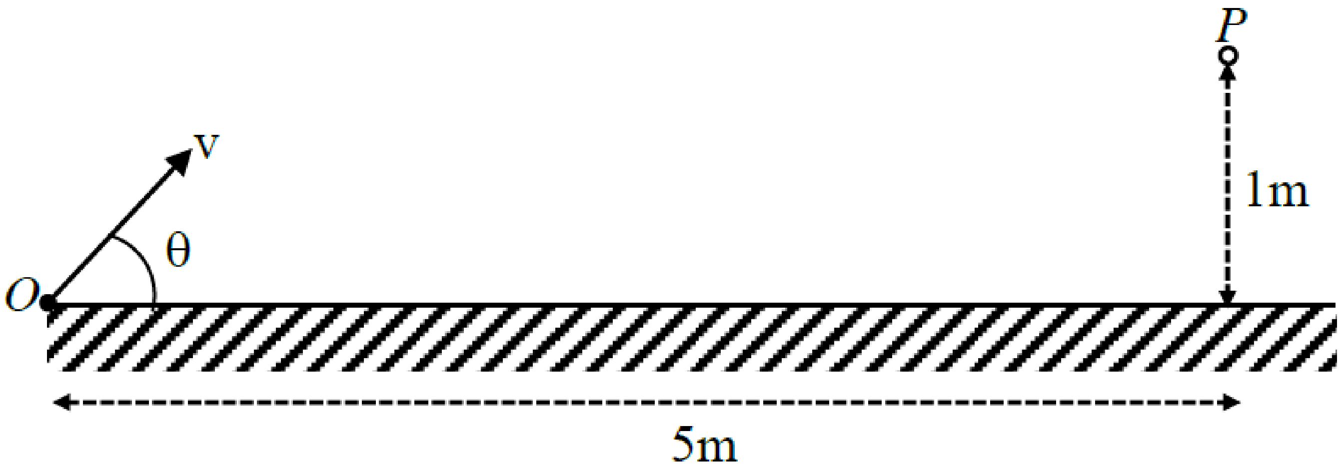
(B) If $\theta = 45°$, the particle reaches its maximum height before it reaches P.

(C) If $\theta = 30°$, the particle reaches its maximum height after reaching P.

(D) If $\theta = \tan^{-1}\left(\frac{1}{5}\right)$, then $v = 125\sqrt{g}\ ms^{-1}$

(A,B)

6. A particle is thrown with a speed v from a point O at an angle θ with the horizontal plane such that it passes through the point P at a height of 1 m and horizontal distance of 5 m from O, as shown in the figure. If acceleration due to gravity is g ms^{-2}, then the correct statement (s) is/are :



$$u^2 = \frac{25}{2} \cdot g \frac{[1+\tan^2\theta]}{[5\tan\theta - 1]}$$

$$\frac{R}{2} = \frac{25}{2} \frac{(1+\tan^2\theta)}{(5\tan\theta - 1)} \sin\theta\cos\theta$$

(A) If θ = 45°, then $v = \frac{5\sqrt{g}}{2} ms^{-1}$

$$u^2 = \frac{25}{2} \cdot g \left(\frac{1+1}{5-1}\right) = \frac{25}{\cancel{2}} \cdot g \frac{(\cancel{2})}{4}$$

(B) If $\theta = 45°$, the particle reaches its maximum height before it reaches P.

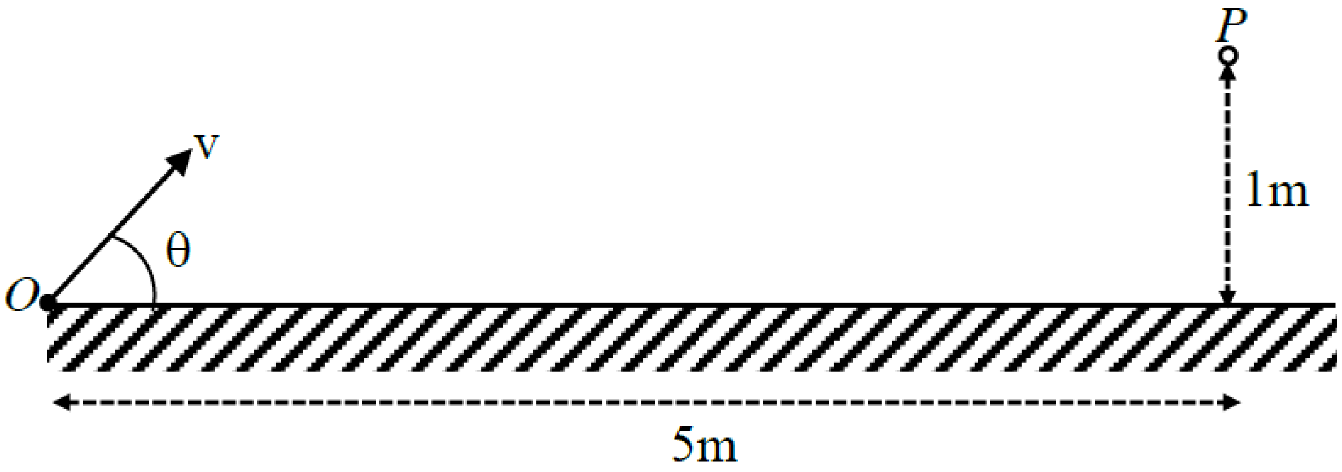
$$u^2 = \frac{25g}{4}$$

(C) If $\theta = 30°$, the particle reaches its maximum height after reaching P.

$$u = \frac{5}{2}\sqrt{g}$$

(D) If $\theta = \tan^{-1}\left(\frac{1}{5}\right)$, then v = $125\sqrt{g}$ ms^{-1}

6. A particle is thrown with a speed v from a point O at an angle θ with the horizontal plane such that it passes through the point P at a height of 1 m and horizontal distance of 5 m from O, as shown in the figure. If acceleration due to gravity is g ms^{-2}, then the correct statement (s) is/are :



$$u^2 = \frac{25}{2} \cdot g \frac{[1+\tan^2\theta]}{[5\tan\theta-1]}$$

$$\frac{R}{2} = \frac{25}{2} \frac{(1+\tan^2\theta)}{(5\tan\theta-1)} \sin\theta\cos\theta$$

$$\frac{R}{2} = \frac{25}{2} \frac{(2)}{(4)} \cdot \frac{1}{\cancel{2}} = \frac{25}{8} = 3.125$$

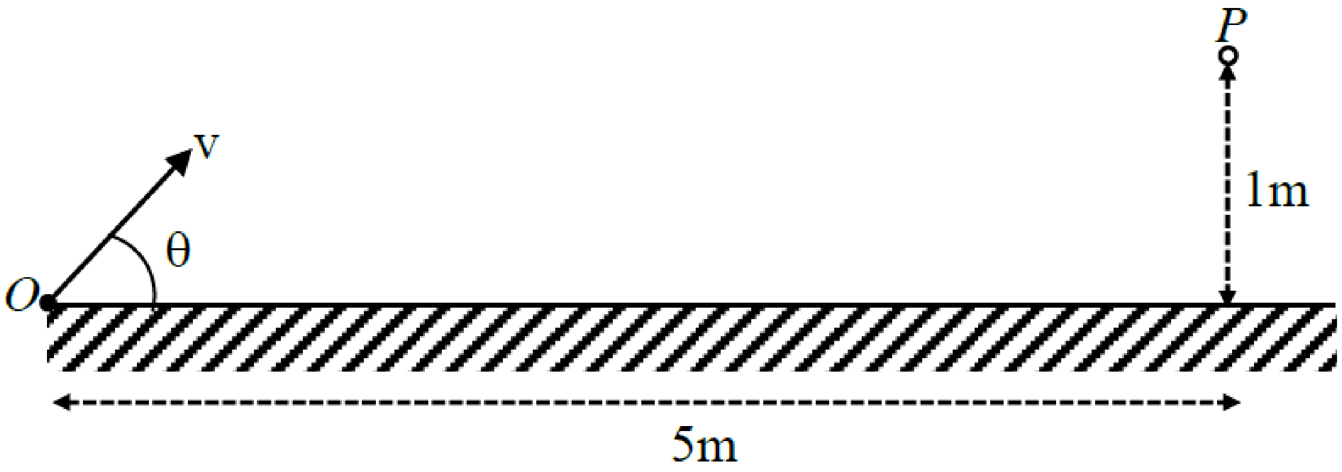
(A) If θ = 45°, then $v = \frac{5\sqrt{g}}{2} ms^{-1}$

(B) If θ = 45°, the particle reaches its maximum height before it reaches P.

(C) If θ = 30°, the particle reaches its maximum height after reaching P.

(D) If $\theta = \tan^{-1}\left(\frac{1}{5}\right)$, then v = $125\sqrt{g}$ ms^{-1}

6. A particle is thrown with a speed v from a point O at an angle θ with the horizontal plane such that it passes through the point P at a height of 1 m and horizontal distance of 5 m from O, as shown in the figure. If acceleration due to gravity is g ms^{-2}, then the correct statement (s) is/are :



$$u^2 = \frac{25}{2} \cdot g \frac{[1+\tan^2\theta]}{[5\tan\theta - 1]}$$

$$\frac{R}{2} = \frac{25}{2} \frac{(1+\tan^2\theta)}{(5\tan\theta - 1)} \sin\theta \cos\theta$$

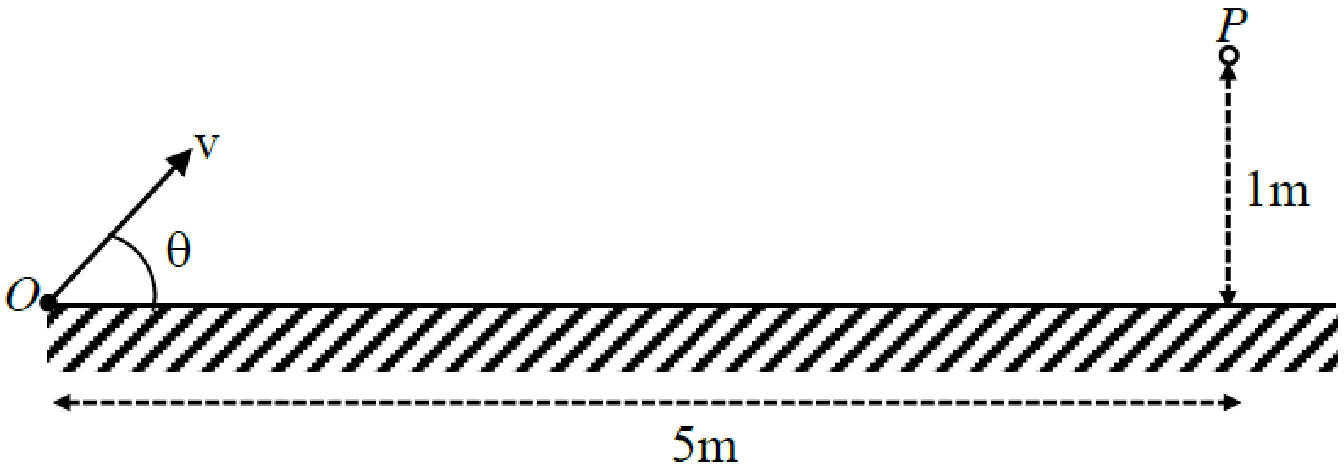
(A) If θ = 45°, then $v = \frac{5\sqrt{g}}{2} ms^{-1}$

(B) If $\theta = 45°$, the particle reaches its maximum height before it reaches P.

(C) If $\theta = 30°$, the particle reaches its maximum height after reaching P.

(D) If $\theta = \tan^{-1}\left(\frac{1}{5}\right)$, then $v = 125\sqrt{g}\ ms^{-1}$

6. A particle is thrown with a speed v from a point O at an angle θ with the horizontal plane such that it passes through the point P at a height of 1 m and horizontal distance of 5 m from O, as shown in the figure. If acceleration due to gravity is g ms^{-2}, then the correct statement (s) is/are :



(A) If θ = 45°, then $v = \frac{5\sqrt{g}}{2} ms^{-1}$

(B) If θ = 45°, the particle reaches its maximum height before it reaches P.

(C) If θ = 30°, the particle reaches its maximum height after reaching P.

(D) If $\theta = \tan^{-1}\left(\frac{1}{5}\right)$, then v = $125\sqrt{g}$ m

$u^2 = \frac{25}{2} \cdot g \frac{[1+\tan^2\theta]}{[5\tan\theta - 1]}$

$\frac{R}{2} = \frac{25}{2} \frac{(1+\tan^2\theta)}{(5\tan\theta - 1)} \sin\theta\cos\theta$

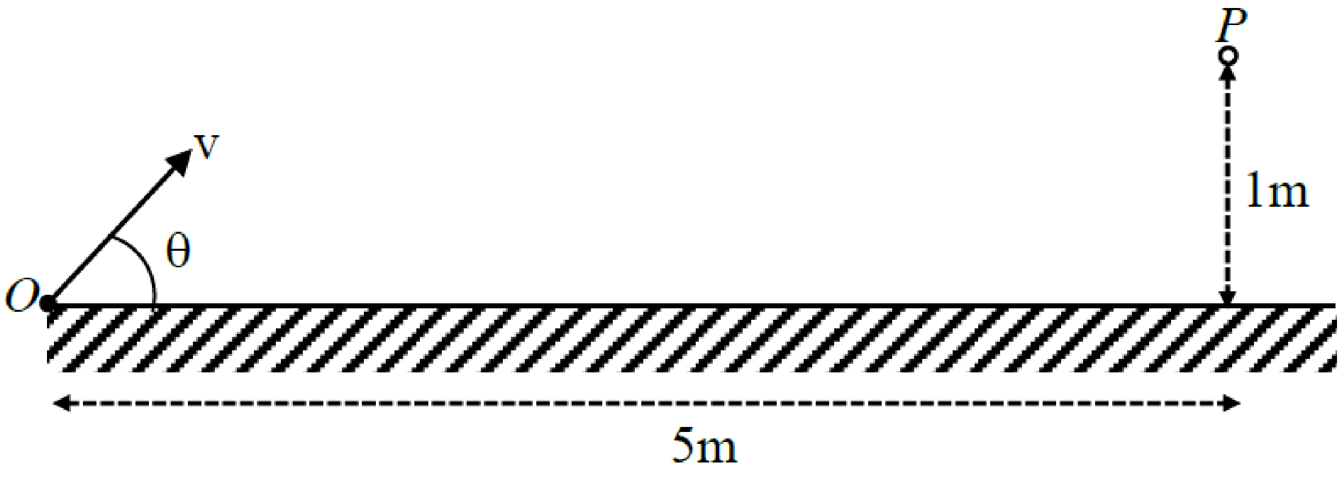
$\frac{R}{2} = \frac{25}{\cancel{2}}\left[\frac{1+3}{5\sqrt{3}-1}\right]\frac{1}{2}\cdot\frac{\sqrt{3}}{2} = \frac{25}{2}\frac{\sqrt{3}}{(5\sqrt{3}-1)}$

$= \frac{25}{2}\frac{\sqrt{3}}{(5\sqrt{3}-1)}\frac{(5\sqrt{3}+1)}{(5\sqrt{3}+1)}$

$= \frac{25[15+\sqrt{3}]}{2[74]} =$

$= \frac{25(16.7)}{2(74)} = 2.8$

6. A particle is thrown with a speed v from a point O at an angle θ with the horizontal plane such that it passes through the point P at a height of 1 m and horizontal distance of 5 m from O, as shown in the figure. If acceleration due to gravity is g ms^{-2}, then the correct statement (s) is/are :



$$u^2 = \frac{25}{2} \cdot g \frac{[1+\tan^2\theta]}{[5\tan\theta - 1]}$$

$$u^2 = \frac{25}{2} \cdot g \frac{(1+\frac{1}{25})}{(5 \cdot \frac{1}{5} - 1)} = \infty$$

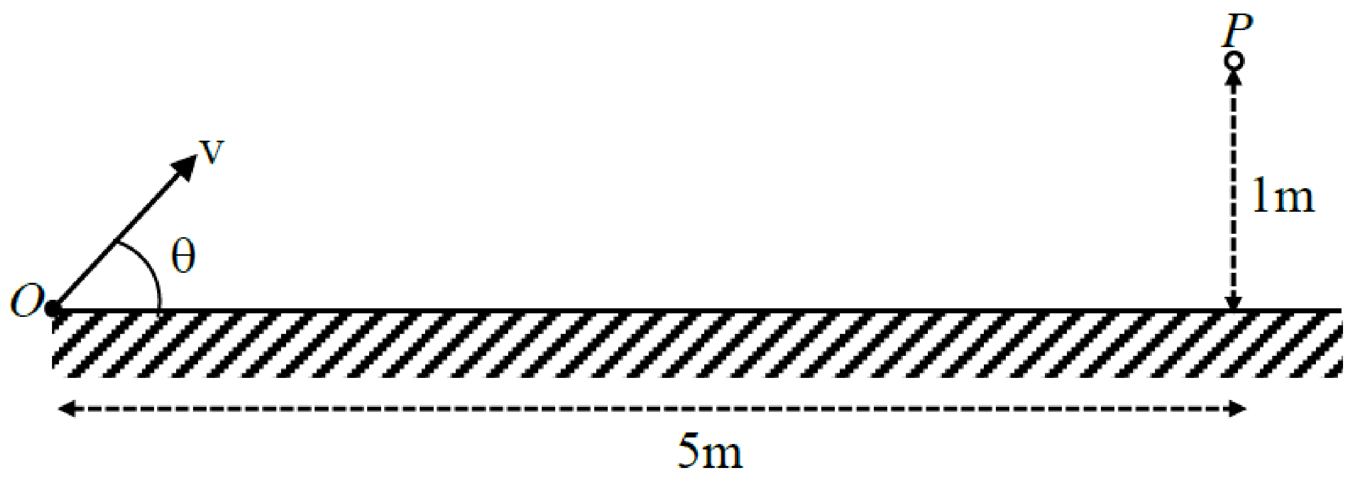
(A) If θ = 45°, then $v = \frac{5\sqrt{g}}{2} ms^{-1}$

(B) If $\theta = 45°$, the particle reaches its maximum height before it reaches P.

(C) If $\theta = 30°$, the particle reaches its maximum height after reaching P.

(D) If $\theta = \tan^{-1}\left(\frac{1}{5}\right)$, then v = $125\sqrt{g}\ ms^{-1}$

6. A particle is thrown with a speed v from a point O at an angle θ with the horizontal plane such that it passes through the point P at a height of 1 m and horizontal distance of 5 m from O, as shown in the figure. If acceleration due to gravity is g ms^{-2}, then the correct statement (s) is/are :

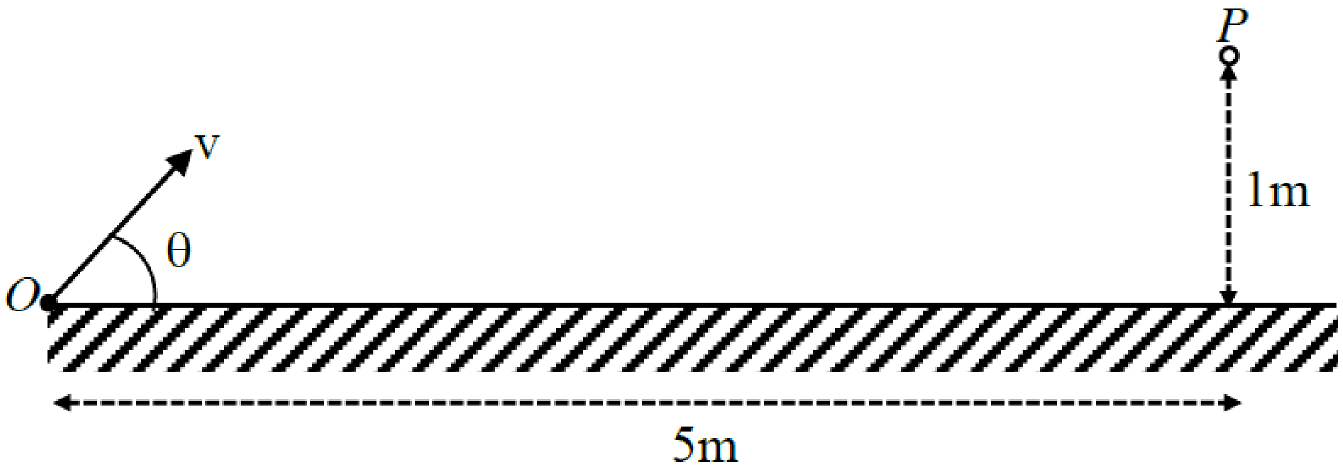


$$y = x\tan\theta - \frac{1}{2}\frac{gx^2}{u^2\cos^2\theta} = x\tan\theta - \frac{1}{2}\frac{gx^2}{u^2}\sec^2\theta = x\tan\theta - \frac{1}{2}\frac{gx^2}{u^2}(1+\tan^2\theta)$$

$$\frac{1}{2}\frac{gx^2}{u^2}(1+\tan^2\theta) = x\tan\theta - y \Rightarrow u^2 = \frac{gx^2(1+\tan^2\theta)}{2[x\tan\theta - y]}$$

$$u^2 = \frac{g.\,25.\,[1+\tan^2\theta]}{2[5\tan\theta - 1]} = \frac{25}{2}.\, g\frac{[1+\tan^2\theta]}{[5\tan\theta-1]}$$

6. A particle is thrown with a speed v from a point O at an angle θ with the horizontal plane such that it passes through the point P at a height of 1 m and horizontal distance of 5 m from O, as shown in the figure. If acceleration due to gravity is g ms^{-2}, then the correct statement (s) is/are :



$$u^2 = \frac{25}{2} \cdot g \frac{[1+\tan^2\theta]}{[5\tan\theta - 1]}$$

$$\frac{R}{2} = \frac{25}{2} \frac{(1+\tan^2\theta)}{(5\tan\theta - 1)} \sin\theta\cos\theta$$

$$R = u^2 \frac{\sin 2\theta}{g}$$

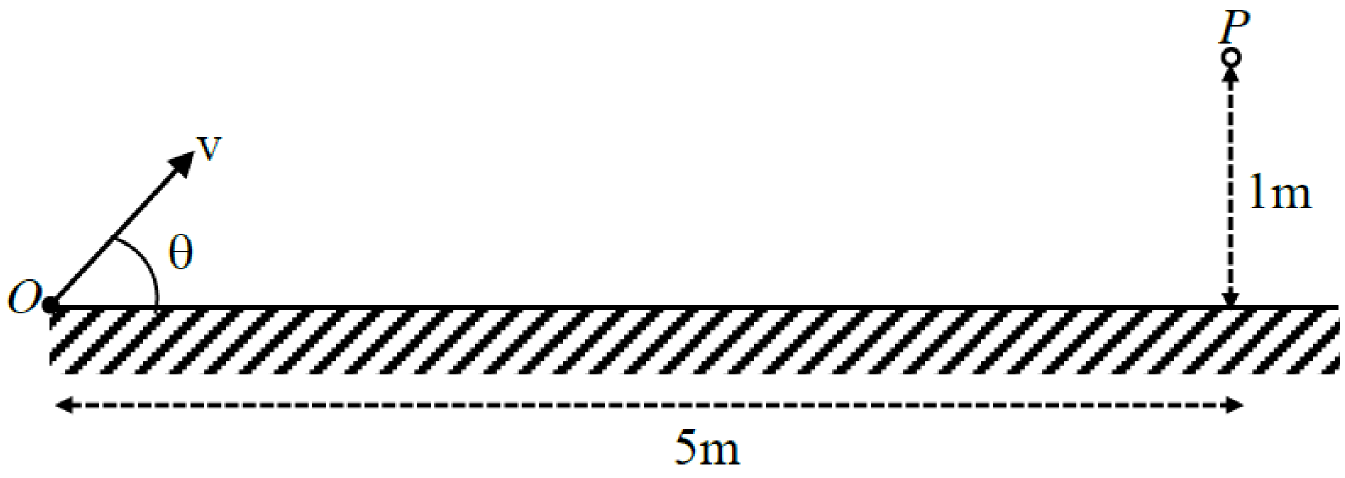
$$\frac{R}{2} = \frac{u^2 \cdot \cancel{2}\sin\theta\cos\theta}{\cancel{2} \cdot g}$$

$$\frac{R}{2} = \frac{25}{2} \cdot \frac{\cancel{g}[1+\tan^2\theta]}{[5\tan\theta - 1]} \frac{\sin\theta\cos\theta}{\cancel{g}}$$

6. A particle is thrown with a speed v from a point O at an angle θ with the horizontal plane such that it passes through the point P at a height of 1 m and horizontal distance of 5 m from O, as shown in the figure. If acceleration due to gravity is g ms^{-2}, then the correct statement (s) is/are :

(A) If $\theta = 45^\circ$, then $v = \frac{5\sqrt{g}}{2} ms^{-1}$

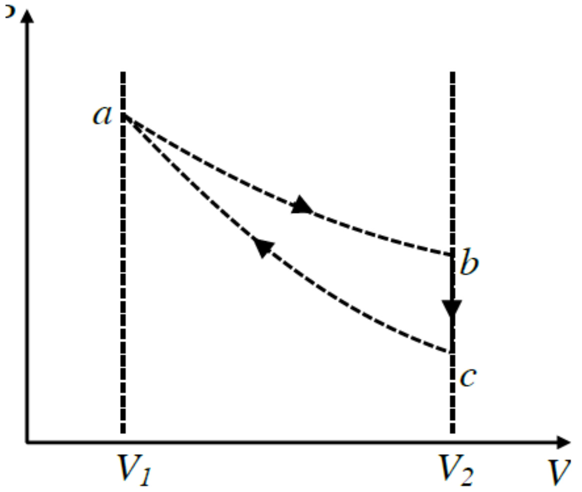
(B) If $\theta = 45^\circ$, the particle reaches its maximum height before it reaches P.



$$u^2 = \frac{25g(1+\tan^2\theta)}{2(5\tan\theta - 1)}$$

$$R = \frac{25(1+\tan^2\theta)\sin\theta\cos\theta}{g(5\tan\theta-1)}$$

7. A quasi-static cycle of a monoatomic ideal gas contains an isothermal process (***ab***), followed by an isochoric process (***bc***) and an adiabatic process (***ca***) as shown in the figure. The volumes of the gas are V_1 and V_2 at ***a*** and ***b***, respectively. If the cycle has heat input Q_{in} and output Q_{out}, then the efficiency of the cycle is defined as $\eta = \frac{Q_{in} - Q_{out}}{Q_{in}}$. The correct statement(s) is/are: [Given : ln 2 ≈ 0.7]



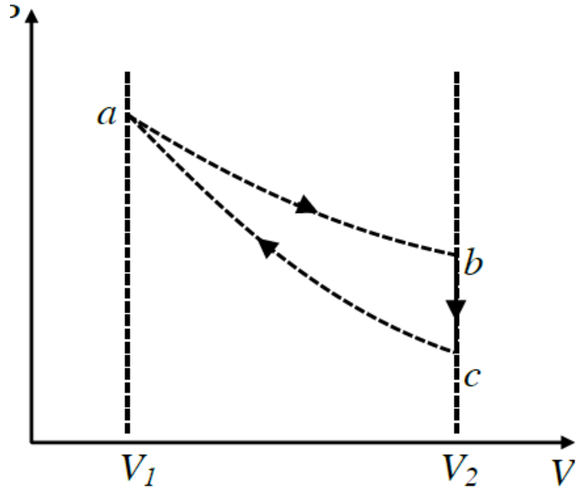
(A) If V_2/V_1=8, the heat released in the process ***bc*** is smaller than the heat absorbed in the process ***ab***.

(B) For a given value of V_2/V_1, η does not depend on the temperature of the isothermal process

(C) If $V_2/V_1 = 8$, then the temperature of the gas at ***a*** is 4 times the temperature of the gas at ***c***.

(D) If $V_2/V_1 = 8$, then the pressure of the gas at ***a*** is 4 times the pressure of the gas at ***b***.

7. A quasi-static cycle of a monoatomic ideal gas contains an isothermal process (***ab***), followed by an isochoric process (***bc***) and an adiabatic process (***ca***) as shown in the figure. The volumes of the gas are V_1 and V_2 at ***a*** and ***b***, respectively. If the cycle has heat input Q_{in} and output Q_{out}, then the efficiency of the cycle is defined as $\eta = \dfrac{Q_{in} - Q_{out}}{Q_{in}}$. The correct statement(s) is/are: [Given : ln 2 ≈ 0.7]



(A) If $V_2/V_1=8$, the heat released in the process ***bc*** is smaller than the heat absorbed in the process ***ab***.

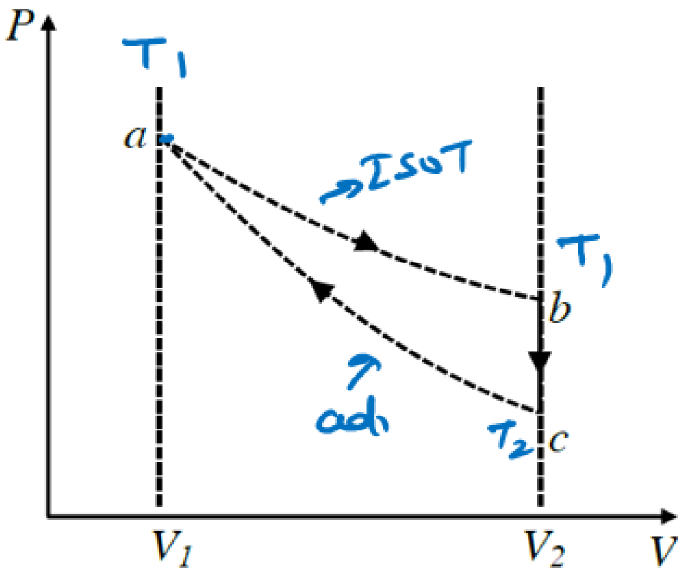
(B) For a given value of V_2/V_1, η does not depend on the temperature of the isothermal process

(C) If $V_2/V_1 = 8$, then the temperature of the gas at ***a*** is 4 times the temperature of the gas at ***c***.

(D) If $V_2/V_1 = 8$, then the pressure of the gas at ***a*** is 4 times the pressure of the gas at ***b***.

(A,B,C)

7. A quasi-static cycle of a monoatomic ideal gas contains an isothermal process (***ab***), followed by an isochoric process (***bc***) and an adiabatic process (***ca***) as shown in the figure. The volumes of the gas are V_1 and V_2 at ***a*** and ***b***, respectively. If the cycle has heat input Q_{in} and output Q_{out}, then the efficiency of the cycle is defined as $\eta = \dfrac{Q_{in} - Q_{out}}{Q_{in}}$. The correct statement(s) is/are: [Given : ln 2 ≈ 0.7]



(A) If $V_2/V_1=8$, the heat released in the process ***bc*** is smaller than the heat absorbed in the process ***ab***.

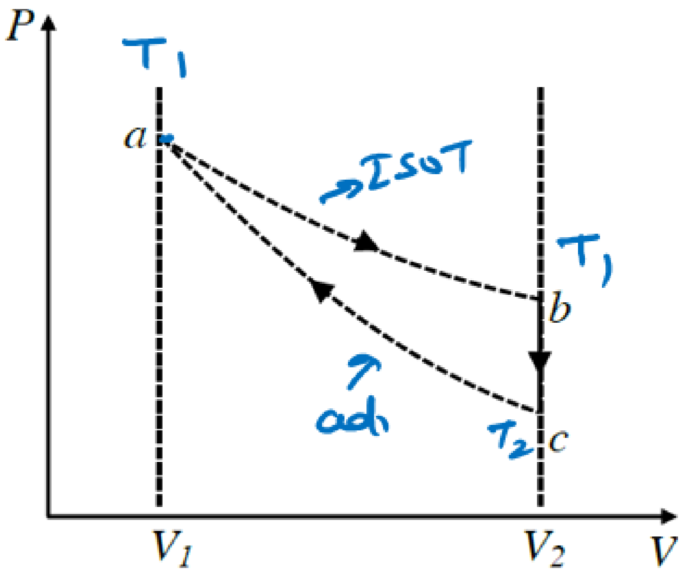
$a \to c \quad T_1 V_1^{2/3} = T_2 V_2^{2/3}$

$Q_{ab} = nRT_1 \ln\frac{V_2}{V_1} = nRT_1 \ln 8 = 3nRT_1(0.7) = 2.1nRT_1$ ✓

$Q_{b\to c} = n \cdot \frac{3}{2}R(T_2 - T_1) = \frac{3}{2}nRT_1\left[\left(\frac{V_1}{V_2}\right)^{2/3} - 1\right]$

$= \frac{3}{2}nRT_1\left[\frac{1}{4} - 1\right] = -\frac{9}{8}nRT_1$

7. A quasi-static cycle of a monoatomic ideal gas contains an isothermal process (***ab***), followed by an isochoric process (***bc***) and an adiabatic process (***ca***) as shown in the figure. The volumes of the gas are V_1 and V_2 at ***a*** and ***b***, respectively. If the cycle has heat input Q_{in} and output Q_{out}, then the efficiency of the cycle is defined as $\eta = \frac{Q_{in} - Q_{out}}{Q_{in}}$. The correct statement(s) is/are: [Given : ln 2 ≈ 0.7]



(B) For a given value of V_2/V_1, η does not depend on the temperature of the isothermal process

$a \to c \quad T_1 V_1^{2/3} = T_2 V_2^{2/3}$

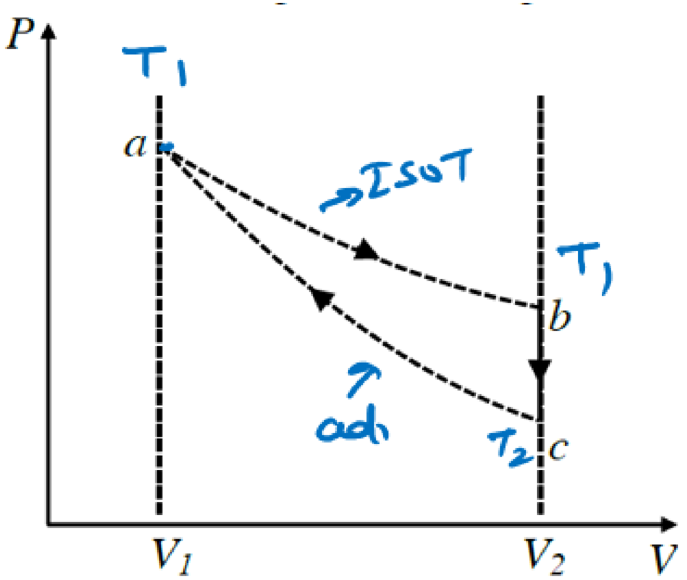
$Q_{ab} = nRT_1 \ln \frac{V_2}{V_1} = nRT_1 \ln 8 = 3nRT_1(0.7) = 2.1nRT_1$ ✓

$Q_{b\to c} = n \cdot \frac{3}{2}R(T_2 - T_1) = \frac{3}{2}nRT_1\left[\left(\frac{V_1}{V_2}\right)^{2/3} - 1\right]$

$= \frac{3}{2}nRT_1\left[\frac{1}{4} - 1\right] = -\frac{9}{8}nRT_1$

$\eta = \frac{2.1nRT_1 - 1.\ nRT_1}{2.1nRT_1}$

7. A quasi-static cycle of a monoatomic ideal gas contains an isothermal process (***ab***), followed by an isochoric process (***bc***) and an adiabatic process (***ca***) as shown in the figure. The volumes of the gas are V_1 and V_2 at ***a*** and ***b***, respectively. If the cycle has heat input Q_{in} and output Q_{out}, then the efficiency of the cycle is defined as $\eta = \dfrac{Q_{in} - Q_{out}}{Q_{in}}$. The correct statement(s) is/are: [Given : ln 2 ≈ 0.7]



(C) If $V_2/V_1 = 8$, then the temperature of the gas at ***a*** is 4 times the temperature of the gas at ***c***.

$a \to c \quad T_1 V_1^{2/3} = T_2 V_2^{2/3}$

$Q_{ab} = nRT_1 \ln \frac{V_2}{V_1} = nRT_1 \ln 8 = 3nRT_1(0.7) = 2.1nRT_1$

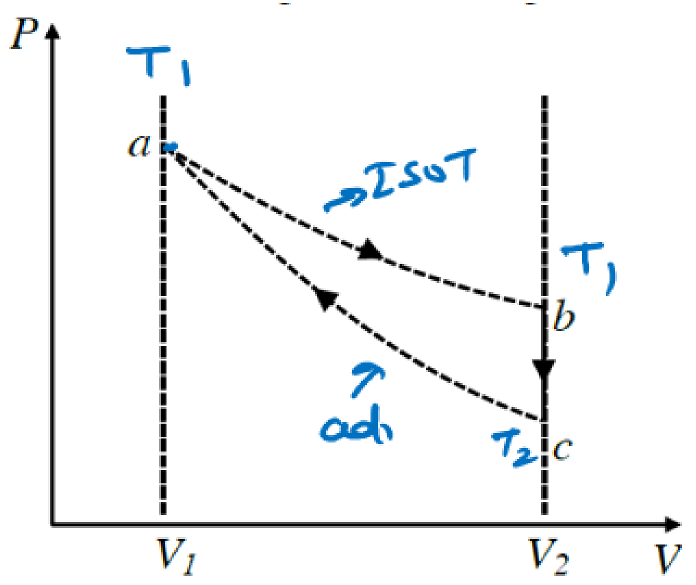
$Q_{b\to c} = n.\frac{3}{2}R(T_2 - T_1) = \frac{3}{2}nRT_1\left[\left(\frac{V_1}{V_2}\right)^{2/3} - 1\right]$

$= \frac{3}{2}nRT_1\left[\frac{1}{4} - 1\right] = -\frac{9}{8}nRT_1$

$\frac{T_1}{T_2} = \left(\frac{V_2}{V_1}\right)^{2/3} = (8)^{2/3} = 2^2 = 4$

$T_1 = 4T_2$

7. A quasi-static cycle of a monoatomic ideal gas contains an isothermal process (***ab***), followed by an isochoric process (***bc***) and an adiabatic process (***ca***) as shown in the figure. The volumes of the gas are V_1 and V_2 at ***a*** and ***b***, respectively. If the cycle has heat input Q_{in} and output Q_{out}, then the efficiency of the cycle is defined as $\eta = \dfrac{Q_{in} - Q_{out}}{Q_{in}}$. The correct statement(s) is/are: [Given : ln 2 ≈ 0.7]



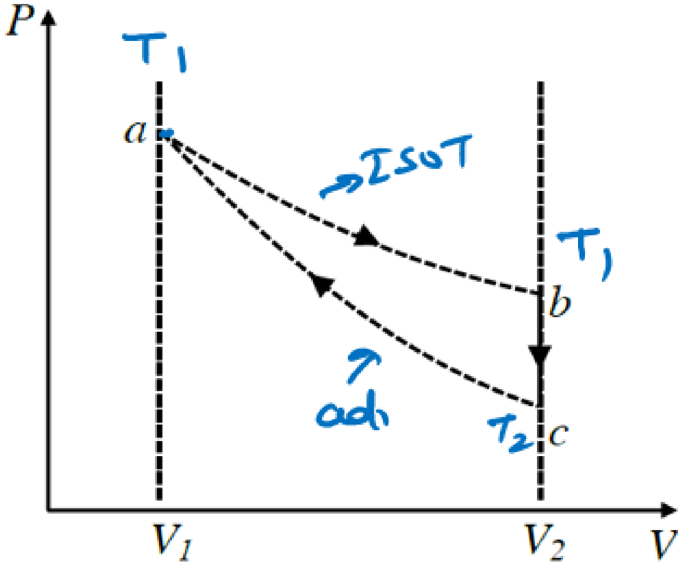
(D) If $V_2/V_1 = 8$, then the pressure of the gas at ***a*** is 4 times the pressure of the gas at ***b***.

a → b isothermal

$PV = K$

$P_1V_1 = P_2V_2$

$\frac{P_1}{P_2} = \frac{V_2}{V_1} = 8 \Rightarrow P_1 = 8P_2$



$a \to c$ ad. $\quad TV^{\gamma-1} = K$

$T_1 V_1^{2/3} = \underline{T_2} V_2^{2/3}$

$\gamma = 1 + \frac{2}{f} = 1 + \frac{2}{3}$

$\gamma - 1 = \frac{2}{3}$

$a \to b$, $dQ = \underset{0}{dU} + dW$

$Q_{ab} = W_{ab} = nRT \ln \frac{V_2}{V}$

$b \to c$, isochoric $\to$ $Q_{b\to c} = nC_V dT = n.\frac{3}{2}R(T_2 - T_1)$

$Q_{b\to c} = \frac{3}{2}nR\left[T_1\left(\frac{V_1}{V_2}\right)^{2/3} - T_1\right] = \frac{3}{2}nRT_1\left[\left(\frac{V_1}{V_2}\right)^{2/3} - 1\right]$

$Q_{c\to a} = 0$ [adiabatic]

8. The electric field associated with an electromagnetic wave travelling in vacuum is given by E0 sin (3y + 4 Z + ωt) i, where ω is the angular frequency. All quantities are in SI units. The correct statement(s) about this wave is/are: [Given: speed of light in vacuum c = 3 × 10^8 ms^{-1}.]

(A) The wave is travelling in $-\frac{1}{5}\left(3\hat{j}+4\hat{k}\right)$ direction.

(B) The magnitude of the wave vector is 0.5 m^{-1}.

(C) The value of ω is 1.5×10^9 rad s^{-1}.

(D) The magnetic field associated with this wave is given by $\frac{E_0}{c}\sin(3y+4z+\omega t)\,(4\hat{j}-3\hat{k})$

The electric field associated with an electromagnetic wave travelling in vacuum is given by E0 sin (3y + 4 Z + ωt) i, where ω is the angular frequency. All quantities are in SI units. The correct statement(s) about this wave is/are: [Given: speed of light in vacuum c = 3 × 10^8 ms^{-1}.]

(A) The wave is travelling in $-\frac{1}{5}\left(3\hat{j}+4\hat{k}\right)$ direction.

(B) The magnitude of the wave vector is 0.5 m^{-1}.

(C) The value of ω is 1.5×10^9 rad s^{-1}.

(D) The magnetic field associated with this wave is given by $\frac{E_0}{c}\sin(3y+4z+\omega t)\,(4\hat{j}-3\hat{k})$

(A,C)

The electric field associated with an electromagnetic wave travelling in vacuum is given by E0 sin (3y + 4 Z + ωt) i, where ω is the angular frequency. All quantities are in SI units. The correct statement(s) about this wave is/are: [Given: speed of light in vacuum c = 3 × 10^8 ms^{-1}.]

(A) The wave is travelling in $-\frac{1}{5}(3\hat{j}+4\hat{k})$ direction.

(B) The magnitude of the wave vector is 0.5 m^{-1}.

(C) The value of ω is 1.5×10^9 rad s^{-1}.

(D) The magnetic field associated with this wave is given by $\frac{E_0}{c}\sin(3y+4z+\omega t)\,(4\hat{j}-3\hat{k})$

$\vec{C} = \vec{E} \times \vec{B}$
$\vec{E} = \vec{B} \times \vec{C}$
$\vec{B} = \vec{C} \times \vec{E}$

$= -\frac{1}{5}(3\hat{j} + 4\hat{k}) \times \hat{i}$
$= -\frac{3}{5}(-k) - \frac{4}{5}\hat{j} = \frac{3}{5}k - \frac{4}{5}\hat{j}$

$\hat{i}$, k, j

Let → E.M. travelling in x dir, E ↑ y, B ↑ z

$E_y = E_0 \sin(kx - \omega t)\hat{j}$, $B_z = B_0 \sin(kx - \omega t)\hat{k}$

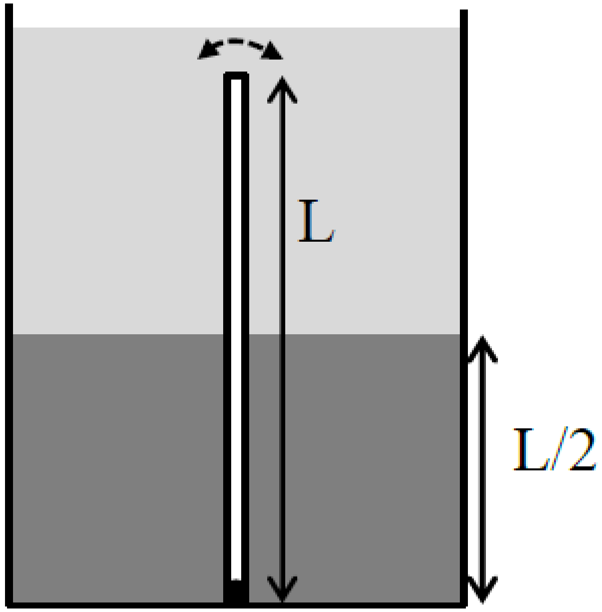
$E_x = E_0 \sin[\underbrace{3y + 4z}{} + \omega t)\hat{x} = E_0 \sin[5(\frac{3}{5}y + \frac{4}{5}z) + \omega t)\hat{i}$

$(kx + \omega t)$

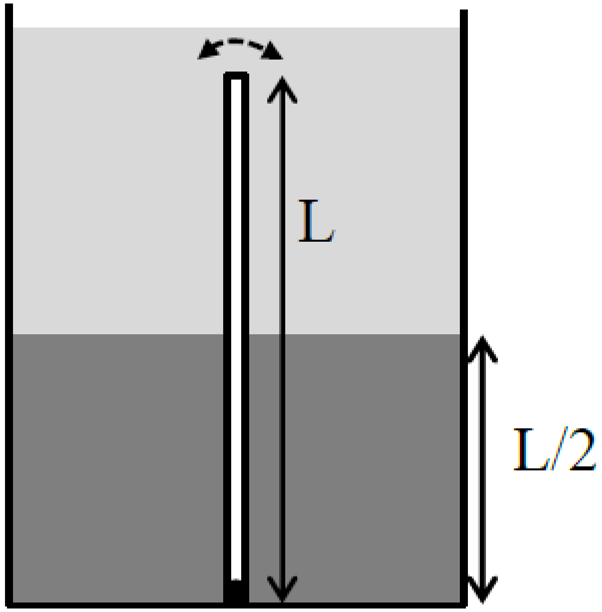
$-(\frac{3}{5}\hat{j} + \frac{4}{5}\hat{k})$

b. 5 . c. $\omega = kc = 5 \times 3 \times 10^8$
$= 15 \times 10^8 \times \frac{10}{10}$

9. A tank contains two immiscible liquids of densities $6\ \rho$ and $2\ \rho$. The higher density liquid is filled up to a height $L / 2$ from the bottom. A thin rod of density ρ and length L is fully immersed and hinged at the bottom so that it can oscillate freely, as shown in the figure. If the rod is slightly disturbed from its equilibrium, the time period of small oscillations is $(2 \pi / \eta) \sqrt{(L / g)}$, where g is the acceleration due to gravity. The value of n is :

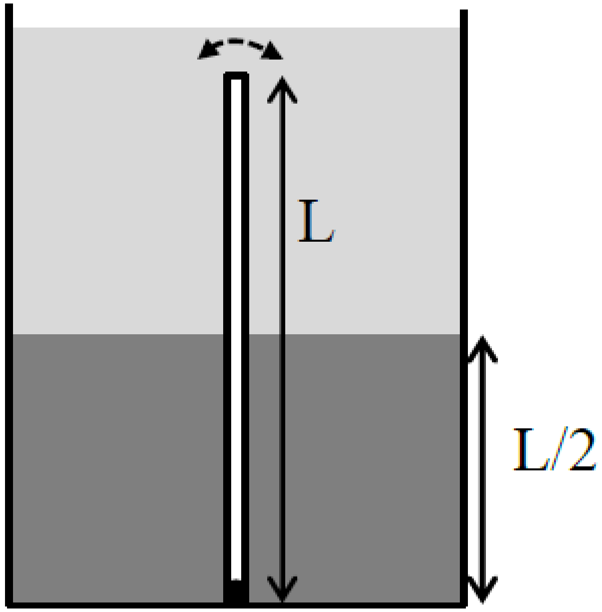


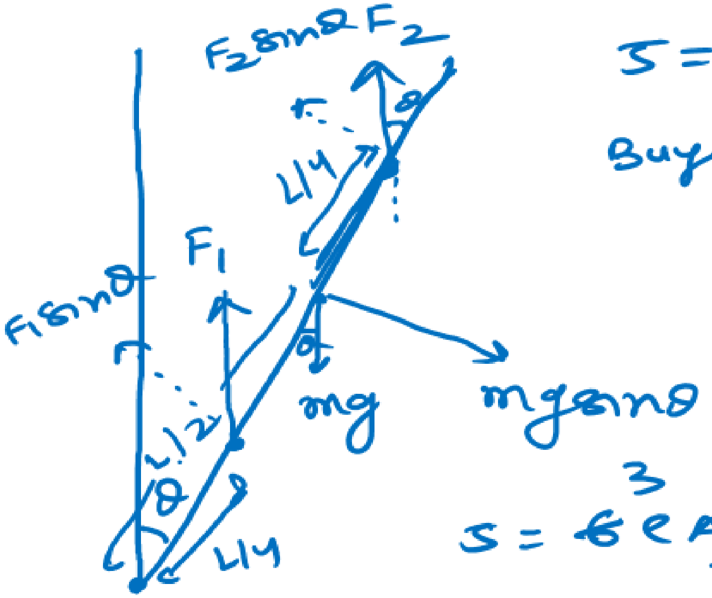
9. A tank contains two immiscible liquids of densities 6 ρ and 2 ρ. The higher density liquid is filled up to a height L / 2 from the bottom. A thin rod of density ρ and length L is fully immersed and hinged at the bottom so that it can oscillate freely, as shown in the figure. If the rod is slightly disturbed from its equilibrium, the time period of small oscillations is (2 π / η) √ (L / g), where g is the acceleration due to gravity. The value of n is :



(1.73)

9. A tank contains two immiscible liquids of densities 6 ρ and 2 ρ. The higher density liquid is filled up to a height L / 2 from the bottom. A thin rod of density ρ and length L is fully immersed and hinged at the bottom so that it can oscillate freely, as shown in the figure. If the rod is slightly disturbed from its equilibrium, the time period of small oscillations is (2 π / η) √ (L / g), where g is the acceleration due to gravity. The value of n is :





$$\tau = F_1\sin\theta\,\frac{L}{4} + F_2\sin\theta\,\frac{3L}{4} - mg\sin\theta\,\frac{L}{2}$$

Buoyant force

$$F_1 = mg = \rho v g = 6\rho \cdot A \cdot \frac{L}{2} \cdot g$$

$$F_2 = 2\rho \cdot A \cdot \frac{L}{\cancel{2}} \cdot g$$

$$m = \rho \cdot A \cdot L$$

$$\tau = \frac{\overset{3}{\cancel{6}}\rho A L g}{\cancel{2}}\,\frac{L}{4}\cdot\theta + \rho A L g\,\frac{3L}{4}\,\theta - \rho A L g\,\frac{L}{2}\,\theta$$

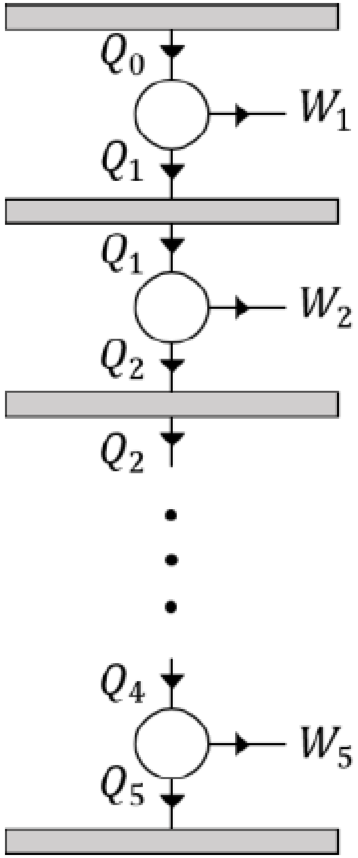
$$\tau = \frac{3}{4}\rho A L^2 g\theta + \frac{3}{4}\rho A L^2 g\theta - \frac{1}{2}\rho A L^2 g\theta = \rho A L^2 g\theta$$

$$I\alpha = \frac{1}{3} m L^2 \alpha = \frac{1}{3}\cdot\cancel{\rho} A L \cdot L^2 \alpha = \cancel{\rho}\,\cancel{A}\,\cancel{L^2}\, g\theta \Rightarrow \alpha = \frac{3g\theta}{L}$$

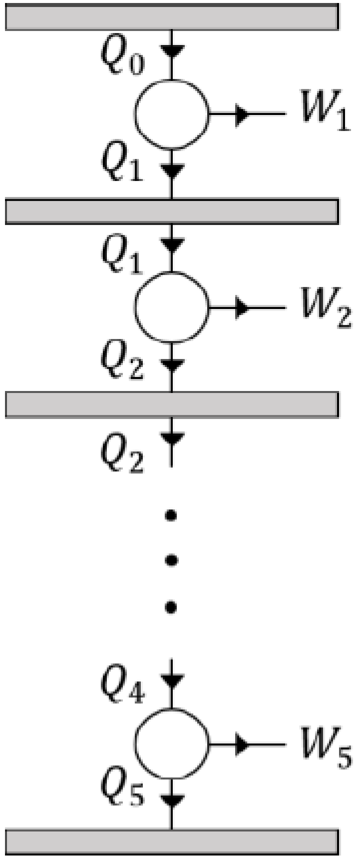
$$\alpha = \omega^2\theta \qquad \omega = \sqrt{\frac{3g}{L}} \qquad T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{L}{3g}}$$

$$n = \sqrt{3} = 1.732$$

10. As shown in the figure, five Carnot engines, each with efficiency η and same number of cycles per unit time, are operating between six heat reservoirs. The amount of heat released per cycle by one engine is completely absorbed by the next engine. Consider Q_0 to be the amount of heat absorbed per cycle by the first engine and W as the amount of total work done by all the engines per cycle, then the net efficiency of the system is found to be $\eta_{net} = (W / Q_0) = (211/243)$. The value of η is:

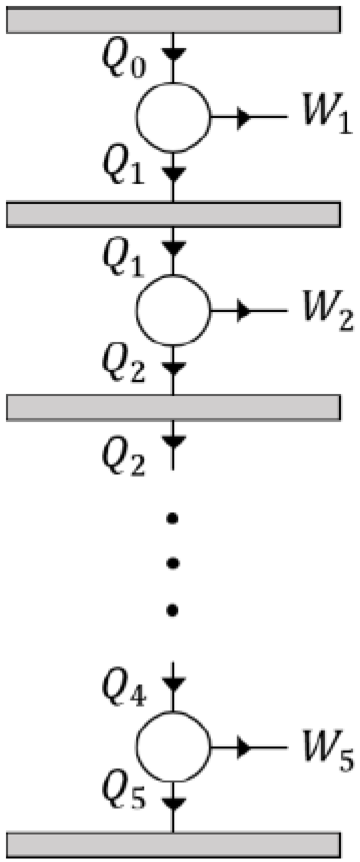


10. As shown in the figure, five Carnot engines, each with efficiency η and same number of cycles per unit time, are operating between six heat reservoirs. The amount of heat released per cycle by one engine is completely absorbed by the next engine. Consider Q_0 to be the amount of heat absorbed per cycle by the first engine and W as the amount of total work done by all the engines per cycle, then the net efficiency of the system is found to be $\eta_{net} = (W / Q_0) = (211/ 243)$. The value of η is:



(0.33)

10. As shown in the figure, five Carnot engines, each with efficiency η and same number of cycles per unit time, are operating between six heat reservoirs. The amount of heat released per cycle by one engine is completely absorbed by the next engine. Consider Q_0 to be the amount of heat absorbed per cycle by the first engine and W as the amount of total work done by all the engines per cycle, then the net efficiency of the system is found to be $\eta_{net} = (W / Q_0) = (211/243)$. The value of η is:



T_0
1
xT_0
2
x^2T_0
3
x^3T_0
4
x^4T_0
5
x^5T_0

$$\eta = 1 - \frac{T_c}{T_H}$$

$$\frac{T_c}{T_H} = 1 - \eta = x$$

$$T_c = xT_H$$

$$\eta_{net} = 1 - \frac{x^5 \cancel{T_0}}{\cancel{T_0}} = \frac{211}{243}$$

$$1 - x^5 = \frac{211}{243}$$

$$x^5 = 1 - \frac{211}{243} = \frac{32}{243}$$

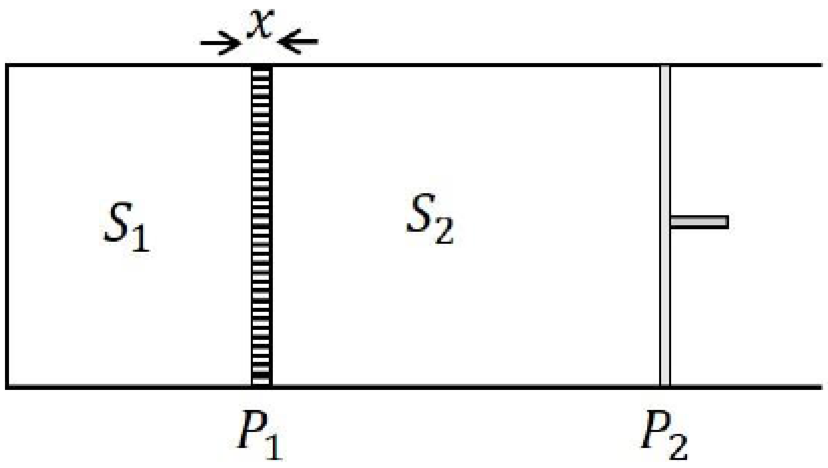
$$x = \frac{2}{3}$$

$$1 - \eta = \frac{2}{3}$$

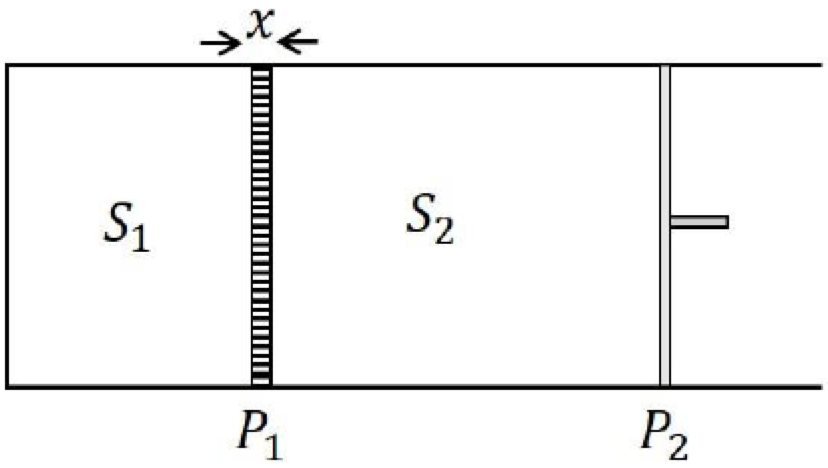
$$\eta = 1 - \frac{2}{3} = \frac{1}{3}$$

$$= 0.33$$

11. As shown in the figure, an insulated container is fitted with a thermally conducting but immovable partition (P_1) and a freely movable but thermally insulated piston (P_2). The partition P_1 with thermal conductivity K, cross sectional area A and width x divides the container into two sections, S_1 and S_2, each containing one mole of a monoatomic gas. The piston P_2 moves freely such that the gas in S_2 is always at the atmospheric pressure. Initially, the difference between the temperatures of S_1 and S_2 is ΔT_0. The time it takes for the temperature difference to become $\Delta T_0 / 2$ is nxR/KA, where R is the universal gas constant. The value of n is: [Given: $ln2 = 0.7$]

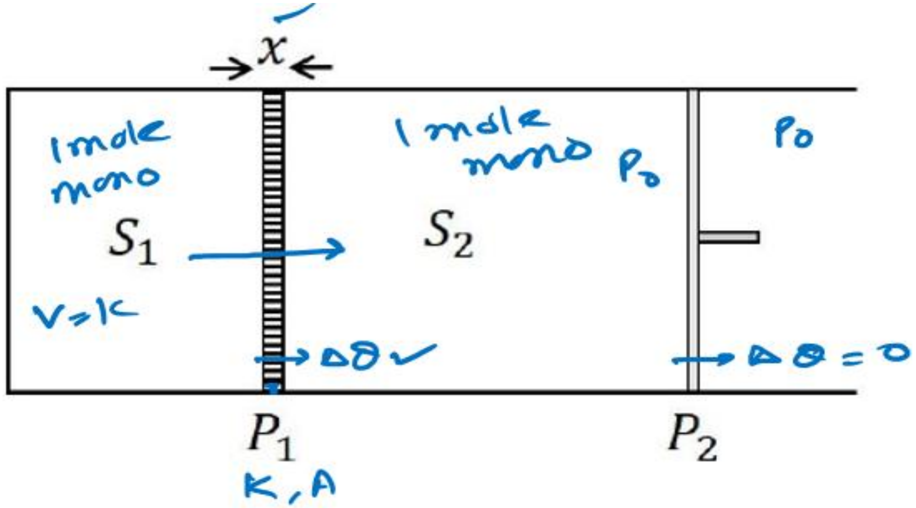


11. As shown in the figure, an insulated container is fitted with a thermally conducting but immovable partition (P_1) and a freely movable but thermally insulated piston (P_2). The partition P_1 with thermal conductivity K, cross sectional area A and width x divides the container into two sections, S_1 and S_2, each containing one mole of a monoatomic gas. The piston P_2 moves freely such that the gas in S_2 is always at the atmospheric pressure. Initially, the difference between the temperatures of S_1 and S_2 is ΔT_0. The time it takes for the temperature difference to become $\Delta T_0 / 2$ is nxR/KA, where R is the universal gas constant. The value of n is: [Given: $ln2 = 0.7$]



(0.66)

11. As shown in the figure, an insulated container is fitted with a thermally conducting but immovable partition (P_1) and a freely movable but thermally insulated piston (P_2). The partition P_1 with thermal conductivity K, cross sectional area A and width x divides the container into two sections, S_1 and S_2, each containing one mole of a monoatomic gas. The piston P_2 moves freely such that the gas in S_2 is always at the atmospheric pressure. Initially, the difference between the temperatures of S_1 and S_2 is ΔT_0. The time it takes for the temperature difference to become $\Delta T_0 / 2$ is nxR/KA, where R is the universal gas constant. The value of n is: [Given: $ln2 = 0.7$]



$dQ = -C_V dT_1 \Rightarrow dT_1 = -dQ/C_V$ — (1)

$dQ = C_P dT_2 \Rightarrow dT_2 = dQ/C_P$ — (2)

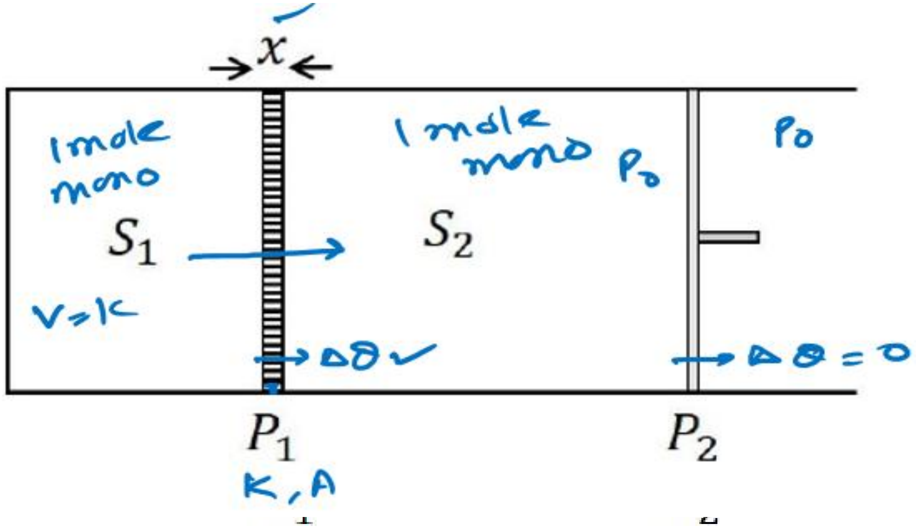
1 − 2, $dT_1 - dT_2 = -\frac{dQ}{C_V} - \frac{dQ}{C_P}$

$d(T_1 - T_2) = -dQ\left(\frac{1}{C_V} + \frac{1}{C_P}\right)$ $C_V = \frac{3R}{2}$, $C_P = \frac{5R}{2}$

$d(\Delta T) = -dQ\left[\frac{2}{3R} + \frac{2}{5R}\right] = -dQ\left[\frac{16}{15R}\right]$

$\frac{dQ}{dt} = KA\frac{\Delta T}{x}$ $d(\Delta T) = -KA\frac{\Delta T}{x}\left[\frac{16}{15R}\right]dt$

11. As shown in the figure, an insulated container is fitted with a thermally conducting but immovable partition (P_1) and a freely movable but thermally insulated piston (P_2). The partition P_1 with thermal conductivity K, cross sectional area A and width x divides the container into two sections, S_1 and S_2, each containing one mole of a monoatomic gas. The piston P_2 moves freely such that the gas in S_2 is always at the atmospheric pressure. Initially, the difference between the temperatures of S_1 and S_2 is ΔT_0. The time it takes for the temperature difference to become $\Delta T_0 / 2$ is nxR/KA, where R is the universal gas constant. The value of n is: [Given: $ln2 = 0.7$]



$$d(\Delta T) = -KA\frac{\Delta T}{x}\left[\frac{16}{15R}\right]dt$$

$$\int_{\Delta T_0}^{\Delta T_0/2}\frac{d(\Delta T)}{\Delta T} = -\frac{KA}{xR}\,\frac{16}{15}\int_0^t dt$$

$$\ln\left(\frac{1}{2}\right) = -\frac{16}{15}\,\frac{KA}{xR}\,t$$

$$\ln 2 = \frac{16}{15}\,\frac{KA}{xR}\,t$$

$$t = \frac{15}{16}\,\frac{xR}{KA}\,0.7$$

$$n = \frac{15}{16}\times\frac{7}{10}$$

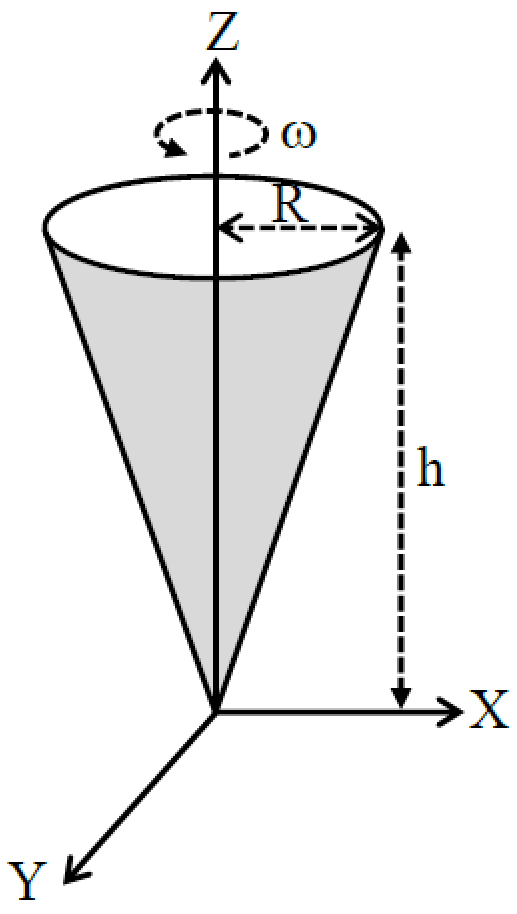
$$n = \frac{105}{160}$$

$$n = 0.656$$

$$n = 0.66$$

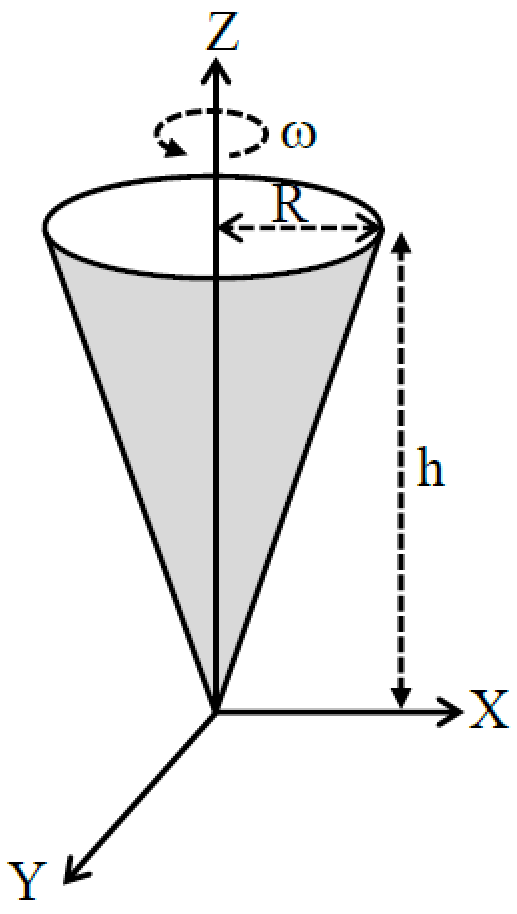
12. A hollow, right circular cone of base radius R and height h, with its tip at the origin is rotating about the Z-axis with an angular velocity ω, as shown in the figure. The cone carries a total charge Q uniformly distributed on its curved surface. The magnitude of magnetic field at a point (0, 0, z),

where $z \gg R$ and $z \gg h$, is $\frac{n\mu_0}{4\pi}\frac{QR^2\omega}{z^3}$. The value of n is:



12. A hollow, right circular cone of base radius R and height h, with its tip at the origin is rotating about the Z-axis with an angular velocity ω, as shown in the figure. The cone carries a total charge Q uniformly distributed on its curved surface. The magnitude of magnetic field at a point (0, 0, z),

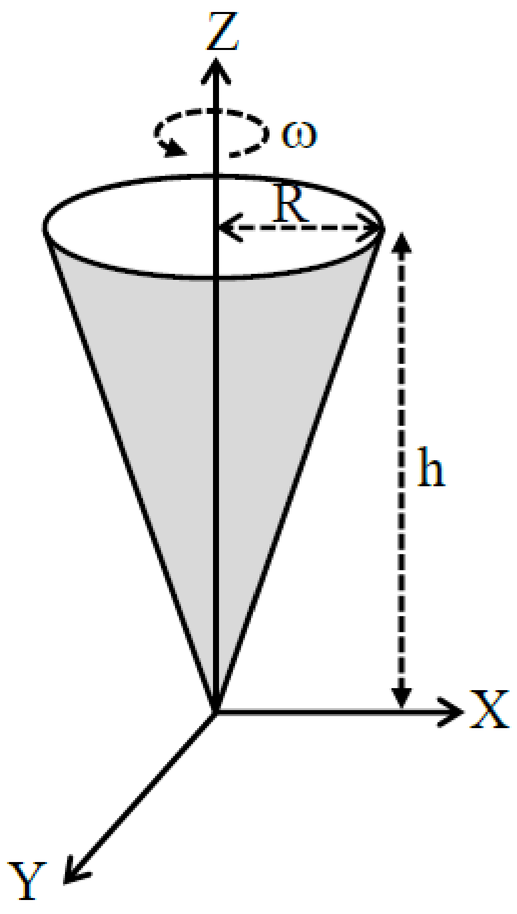
where $z >> R$ and $z >> h$, is $\frac{n\mu_0}{4\pi}\frac{QR^2\omega}{z^3}$. The value of n is:



(0.5)

12. A hollow, right circular cone of base radius R and height h, with its tip at the origin is rotating about the Z-axis with an angular velocity ω, as shown in the figure. The cone carries a total charge Q uniformly distributed on its curved surface. The magnitude of magnetic field at a point (0, 0, z),

where z >> R and z >> h, is $\frac{n\mu_0}{4\pi}\frac{QR^2\omega}{z^3}$. The value of n is:



rotating charge behaves as a mag dipole

$\frac{|\vec{M}|}{L} = \frac{Q}{2m}$

$L = I\omega = \frac{1}{2} mR^2\omega$

$\frac{2|\vec{M}|}{\cancel{m}R^2\omega} = \frac{Q}{2\cancel{m}}$

$|\vec{m}| = \frac{QR^2\omega}{4}$

$B_{axial} = \frac{2KM}{r^3}$

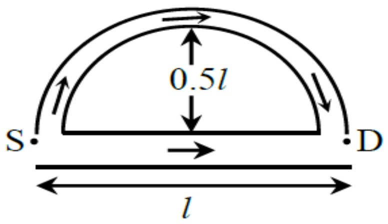
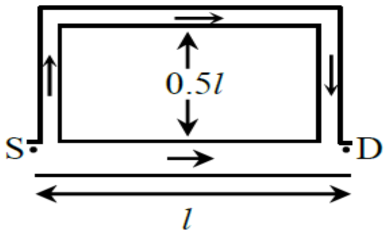
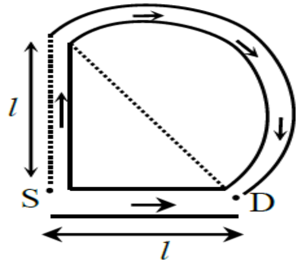
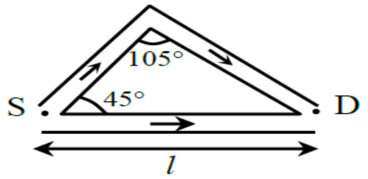
$B = \cancel{2} \cdot \frac{\mu_0}{4\pi} \frac{QR^2\omega}{\cancel{4}_2 z^3}$

$n = \frac{1}{2}$

$n = 0.5$

13. List-I shows four configurations made of straight and semi-circular narrow tubes containing air. A sound wave of wavelength λ = 0.29 m enters these structures at the point S and a sound detector is placed at D. Between the points S and D, the sound travels only through the tubes. **List-II** contains the possible smallest values of l (refer to the figures) for which the detector D records maximum amplitude. Ignore effects of sharp corners. [Given cos(15°) = 0.97]

Choose the option that best describes the match between the entries in **List-I** to those in **List-II**.

	List-I		List-II
(P)		(1)	1.32 m
(Q)		(2)	1.19 m
(R)		(3)	0.51 m
(S)		(4)	0.29 m
		(5)	0.13 m

(A) P→4, Q→3, R→5, S→1

(B) P→4, Q→3, R→1, S→5

(C) P→3, Q→4, R→1, S→2

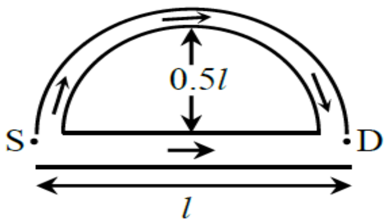
(D) P→3, Q→4, R→5, S→2

13. List-I shows four configurations made of straight and semi-circular narrow tubes containing air. A sound wave of wavelength $\lambda = 0.29$ m enters these structures at the point S and a sound detector is placed at D. Between the points S and D, the sound travels only through the tubes. **List-II** contains the possible smallest values of l (refer to the figures) for which the detector D records maximum amplitude. Ignore effects of sharp corners. [Given cos(15°) = 0.97]

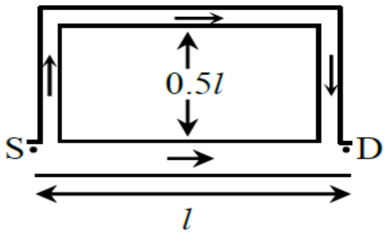
Choose the option that best describes the match between the entries in **List-I** to those in **List-II**.

List-I

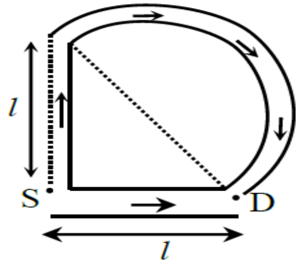
(P)



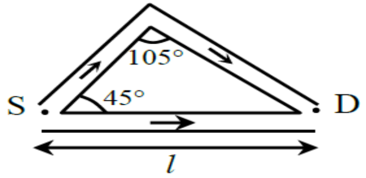
(Q)



(R)



(S)



List-II

(1) 1.32 m

(2) 1.19 m

(3) 0.51 m

(4) 0.29 m

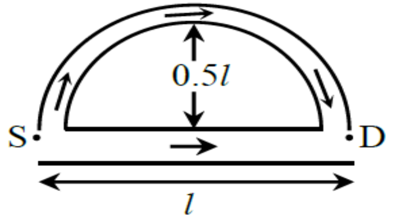
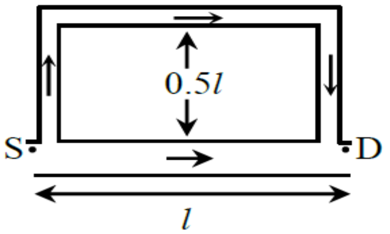
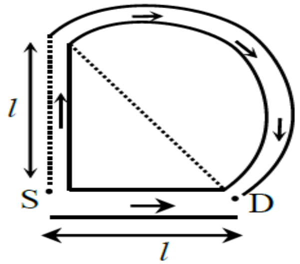
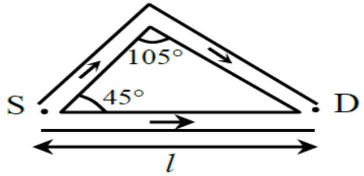
(5) 0.13 m

(A) P→4, Q→3, R→5, S→1

(B) P→4, Q→3, R→1, S→5

(C) P→3, Q→4, R→1, S→2

(D) P→3, Q→4, R→5, S→2

List-I		List-II	
(P)		(1)	1.32 m
(Q)		(2)	1.19 m
(R)		(3)	0.51 m
(S)		(4)	0.29 m
		(5)	0.13 m

(A) P→4, Q→3, R→5, S→1

(B) P→4, Q→3, R→1, S→5

(C) P→3, Q→4, R→1, S→2

(D) P→3, Q→4, R→5, S→2

$\lambda = 0.29\text{ m}$

$\Delta x = n\lambda$

(P) $\pi\left[\frac{l}{2}\right] - l = n\lambda$

$n = 1$

$l\left[\frac{\pi}{2} - 1\right] = \lambda$

$l = \frac{\lambda \, 2}{(\pi - 2)} = \frac{2 \times 0.29}{(3.14 - 2)} = \frac{0.58}{1.14}$

$= 0.508$

(R) $\left(\cancel{l} + \pi\frac{l}{\sqrt{2}}\right) - \cancel{l} = \lambda$

$l = \frac{\lambda\sqrt{2}}{\pi} = \frac{0.29 \times 1.414}{3.14} = \frac{0.41006}{3.14} = 0.130$

14. In the **List-I**, four optical effects are mentioned. The physical phenomena of light which are essential to describe these optical effects are given in **List-II**. Choose the option which describes the correct match between the entries in **List-I** to those in **List-II**.

List-I		**List-II**	
(P)	Colorful sky in north polar region (Aurora Borealis)	(1)	Dispersion and reflection
(Q)	Partially polarized sun light	(2)	Total internal reflection
(R)	Rainbow	(3)	Diffraction
(S)	Dark and bright fringes	(4)	Scattering of light by molecules in the atmosphere
		(5)	Emission of radiation from oxygen and nitrogen atoms excited by charged particles

(A) $P \rightarrow 5$, $Q \rightarrow 4$, $R \rightarrow 1$, $S \rightarrow 3$

(B) $P \rightarrow 4$, $Q \rightarrow 2$, $R \rightarrow 1$, $S \rightarrow 3$

(C) $P \rightarrow 4$, $Q \rightarrow 1$, $R \rightarrow 2$, $S \rightarrow 3$

(D) $P \rightarrow 5$, $Q \rightarrow 4$, $R \rightarrow 1$, $S \rightarrow 2$

14. In the **List-I**, four optical effects are mentioned. The physical phenomena of light which are essential to describe these optical effects are given in **List-II**. Choose the option which describes the correct match between the entries in **List-I** to those in **List-II**.

	List-I		**List-II**
(P)	Colorful sky in north polar region (Aurora Borealis)	(1)	Dispersion and reflection
(Q)	Partially polarized sun light	(2)	Total internal reflection
(R)	Rainbow	(3)	Diffraction
(S)	Dark and bright fringes	(4)	Scattering of light by molecules in the atmosphere
		(5)	Emission of radiation from oxygen and nitrogen atoms excited by charged particles

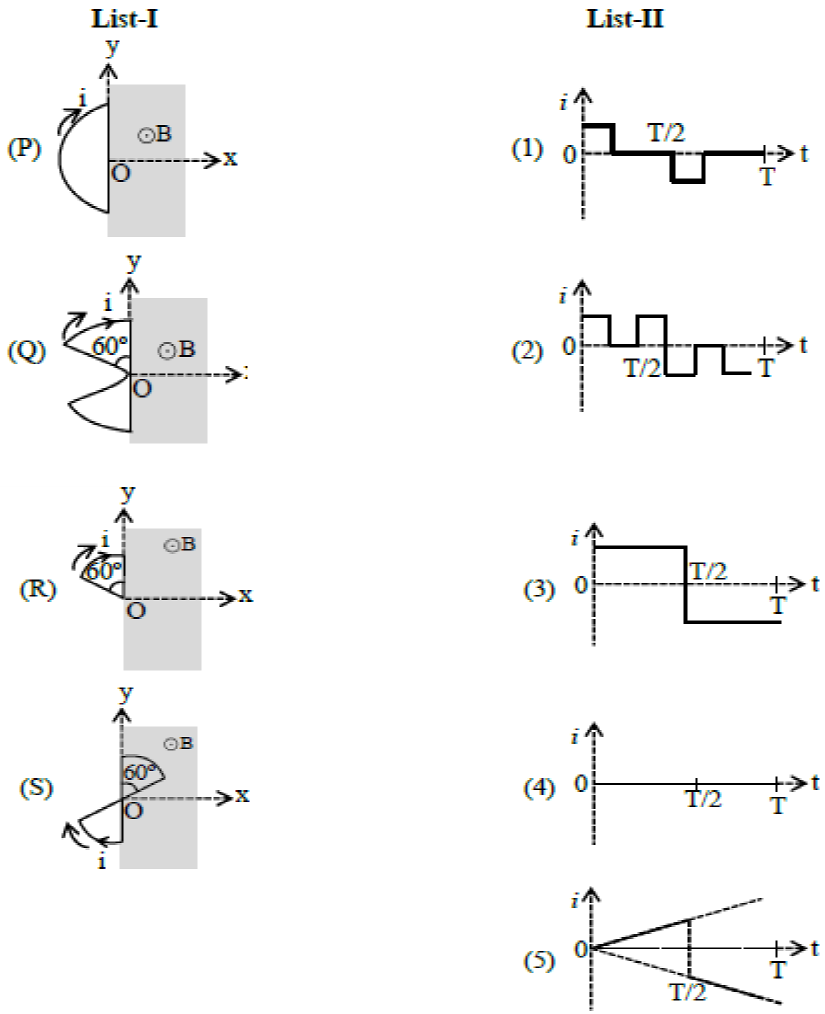
(A) P→5, Q→4, R→1, S→3

(B) P→4, Q→2, R→1, S→3

(C) P→4, Q→1, R→2, S→3

(D) P→5, Q→4, R→1, S→2

15. List-I contains four conducting loops lying in the XY plane, as shown in the figures. The loops are rotating about Z axis passing through the point 0 with time period T in clockwise direction. The region x > 0 contains a uniform magnetic field B in the +z direction. **List-II** contains the qualitative variation of the induced current i(t) for each of these loops. Choose the option which describes the correct match between the entries in **List-I** to those in **List-II**.



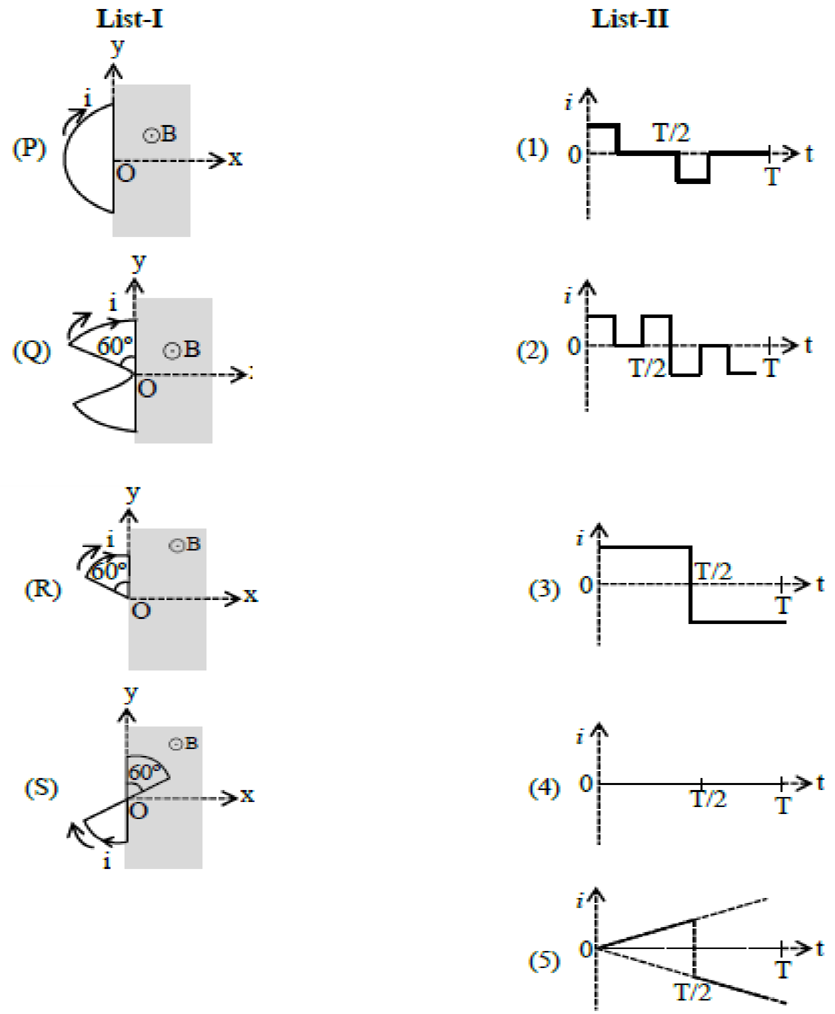
(A) $P \rightarrow 5, Q \rightarrow 4, R \rightarrow 1, S \rightarrow 3$

(B) $P \rightarrow 3, Q \rightarrow 2, R \rightarrow 5, S \rightarrow 4$

(C) $P \rightarrow 3, Q \rightarrow 2, R \rightarrow 1, S \rightarrow 4$

(D) $P \rightarrow 5, Q \rightarrow 1, R \rightarrow 2, S \rightarrow 3$

15. **List-I** contains four conducting loops lying in the XY plane, as shown in the figures. The loops are rotating about Z axis passing through the point 0 with time period T in clockwise direction. The region x > 0 contains a uniform magnetic field B in the +z direction. **List-II** contains the qualitative variation of the induced current i(t) for each of these loops. Choose the option which describes the correct match between the entries in **List-I** to those in **List-II**.



(A) $P \rightarrow 5, Q \rightarrow 4, R \rightarrow 1, S \rightarrow 3$

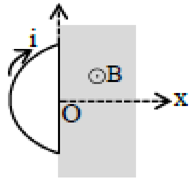
(B) $P \rightarrow 3, Q \rightarrow 2, R \rightarrow 5, S \rightarrow 4$

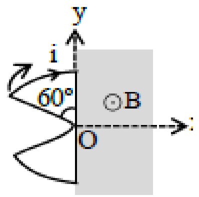
(C) $P \rightarrow 3, Q \rightarrow 2, R \rightarrow 1, S \rightarrow 4$

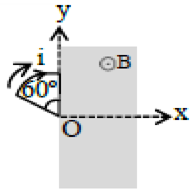
(D) $P \rightarrow 5, Q \rightarrow 1, R \rightarrow 2, S \rightarrow 3$

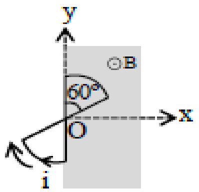
15. List-I contains four conducting loops lying in the XY plane, as shown in the figures. The loops are rotating about Z axis passing through the point 0 with time period T in clockwise direction. The region x > 0 contains a uniform magnetic field B in the +z direction. **List-II** contains the qualitative variation of the induced current i(t) for each of these loops. Choose the option which describes the correct match between the entries in **List-I** to those in **List-II**.

List-I

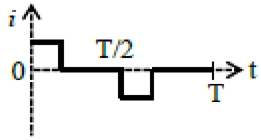
(P)


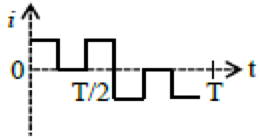
(Q)


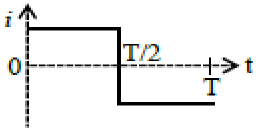
(R)


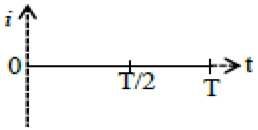
(S)


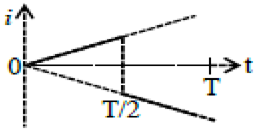
List-II

(1)


(2)


(3)


(4)


(5)


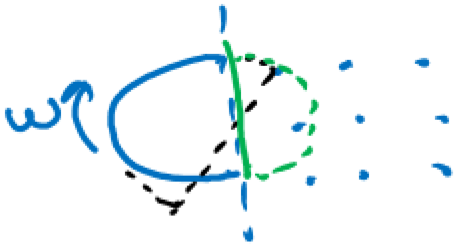
(A) $P \rightarrow 5, Q \rightarrow 4, R \rightarrow 1, S \rightarrow 3$

(B) $P \rightarrow 3, Q \rightarrow 2, R \rightarrow 5, S \rightarrow 4$

(C) $P \rightarrow 3, Q \rightarrow 2, R \rightarrow 1, S \rightarrow 4$

(D) $P \rightarrow 5, Q \rightarrow 1, R \rightarrow 2, S \rightarrow 3$

(P)



$\phi = BA\cos\theta$

$\phi \uparrow$

$e = -\left|\frac{d\phi}{dt}\right|$

$0 \rightarrow 180^\circ$ $\phi \uparrow$ CW $0 \rightarrow T/2$

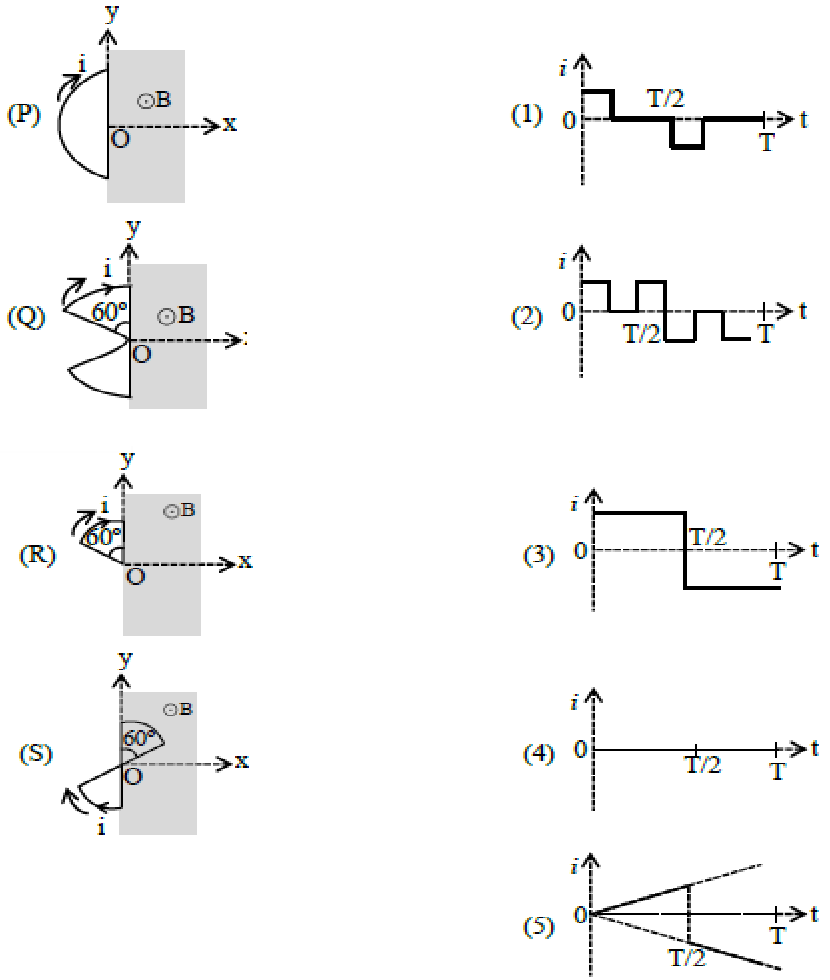
$180 \rightarrow 360^\circ$ $\phi \downarrow$ ACW $T/2 \rightarrow T$

$\phi = B.\frac{1}{2}R\,R\theta = \frac{1}{2}BR^2\omega t$

$e = \frac{d\phi}{dt} = \frac{1}{2}BR^2\omega$ K

15. List-I contains four conducting loops lying in the XY plane, as shown in the figures. The loops are rotating about Z axis passing through the point 0 with time period T in clockwise direction. The region x > 0 contains a uniform magnetic field B in the +z direction. **List-II** contains the qualitative variation of the induced current i(t) for each of these loops. Choose the option which describes the correct match between the entries in **List-I** to those in **List-II**.

List-I List-II



(A) $P \to 5, Q \to 4, R \to 1, S \to 3$

(B) $P \to 3, Q \to 2, R \to 5, S \to 4$

(C) $P \to 3, Q \to 2, R \to 1, S \to 4$

(D) $P \to 5, Q \to 1, R \to 2, S \to 3$

$0° \to 60°$ $\phi \uparrow$ $i = k$

$60° \to \theta$ ϕ const $i = 0$

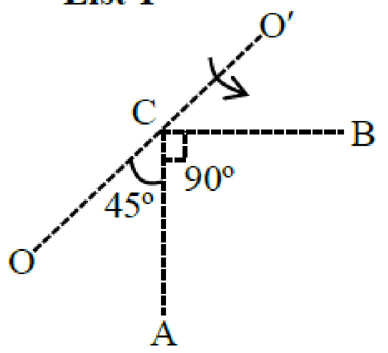
$\theta \to$ other $\phi \downarrow$ i opp

$360° \to T$

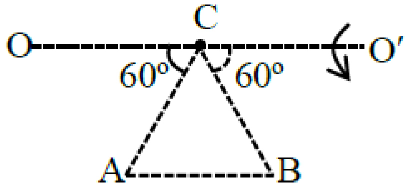
$1° \to \frac{T}{360°} \times 60° = \frac{T}{6}$

16. List-I shows four planar structures made of uniform solid rods each of mass m and length l. In the **List-II** the possible moment of inertia of these structures about an axis OCO’, which lies in the plane of the structures, are given. Choose the option that describes the correct match between the entries in **List-I** to those in **List-II**.

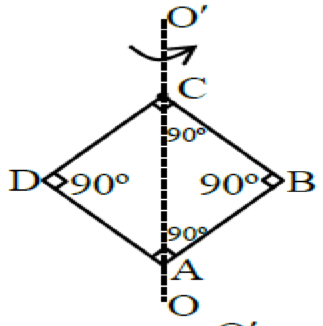
List-I | **List-II**

(P)


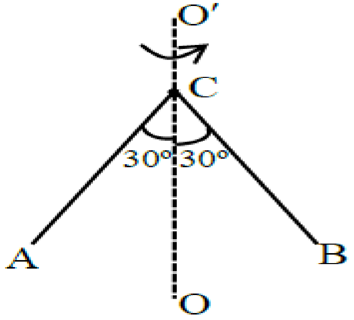
(1) $\frac{5}{4}m\ell^2$

(Q)


(2) $\frac{1}{6}m\ell^2$

(R)


(3) $\frac{1}{12}m\ell^2$

(S)


(4) $\frac{2}{3}m\ell^2$

(5) $\frac{1}{3}m\ell^2$

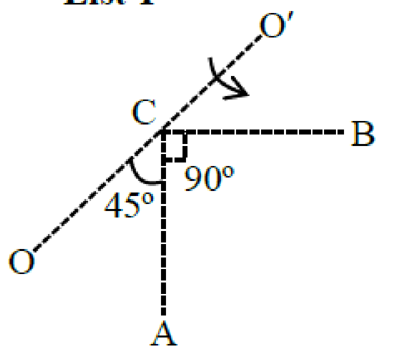
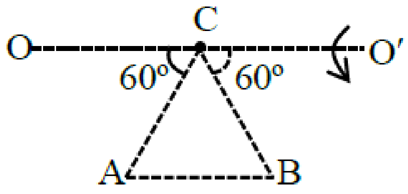
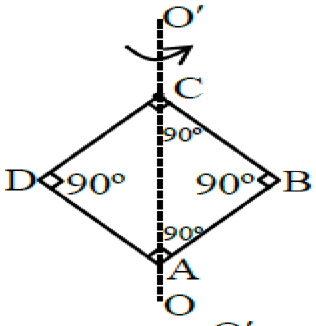
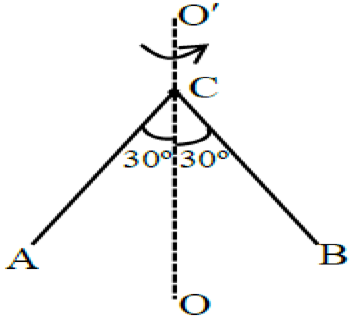
(A) $P \rightarrow 5, Q \rightarrow 1, R \rightarrow 4, S \rightarrow 2$

(B) $P \rightarrow 1, Q \rightarrow 3, R \rightarrow 4, S \rightarrow 2$

(C) $P \rightarrow 5, Q \rightarrow 3, R \rightarrow 2, S \rightarrow 1$

(D) $P \rightarrow 5, Q \rightarrow 4, R \rightarrow 2, S \rightarrow 1$

16. List-I shows four planar structures made of uniform solid rods each of mass m and length l. In the **List-II** the possible moment of inertia of these structures about an axis OCO', which lies in the plane of the structures, are given. Choose the option that describes the correct match between the entries in **List-I** to those in **List-II**.

List-I	List-II
(P) 	(1) $\frac{5}{4}m\ell^2$
(Q) 	(2) $\frac{1}{6}m\ell^2$
(R) 	(3) $\frac{1}{12}m\ell^2$
(S) 	(4) $\frac{2}{3}m\ell^2$

(A) $P \to 5, Q \to 1, R \to 4, S \to 2$

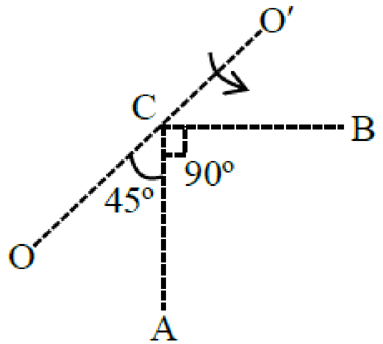
(B) $P \to 1, Q \to 3, R \to 4, S \to 2$

(C) $P \to 5, Q \to 3, R \to 2, S \to 1$

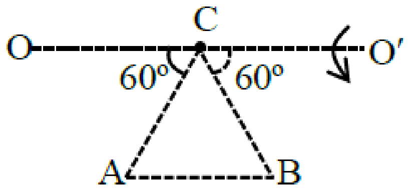
(D) $P \to 5, Q \to 4, R \to 2, S \to 1$

16. List-I shows four planar structures made of uniform solid rods each of mass m and length l. In the **List-II** the possible moment of inertia of these structures about an axis OCO', which lies in the plane of the structures, are given. Choose the option that describes the correct match between the entries in **List-I** to those in **List-II**.

List-I | **List-II**

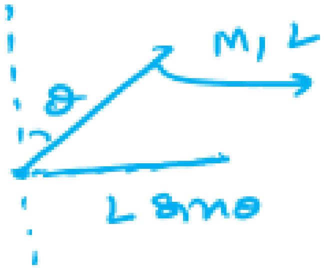
(P) 

(1) $\frac{5}{4}m\ell^2$

(Q) 

(2) $\frac{1}{6}m\ell^2$

P.



$I = \frac{1}{3} m L^2 \sin^2\theta$

$I_{BC} = \frac{1}{3} m l^2 \frac{1}{2} = \frac{1}{6} ml^2$

$I_{AC} = \frac{1}{6} ml^2$

$I = \frac{1}{3} ml^2$

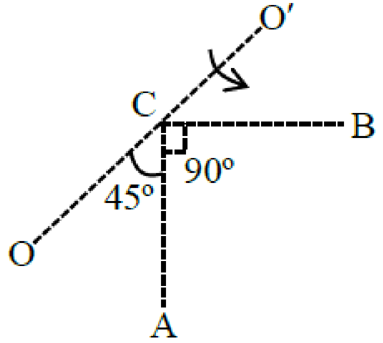
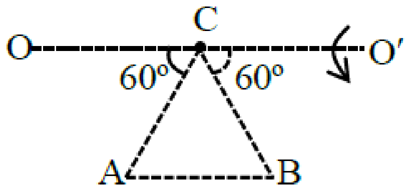
(A) P → 5, Q → 1, R → 4, S → 2

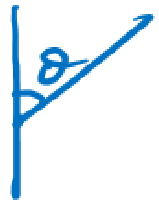
(B) P → 1, Q → 3, R → 4, S → 2

(C) P → 5, Q → 3, R → 2, S → 1

(D) P → 5, Q → 4, R → 2, S → 1

16. List-I shows four planar structures made of uniform solid rods each of mass m and length l. In the **List-II** the possible moment of inertia of these structures about an axis OCO’, which lies in the plane of the structures, are given. Choose the option that describes the correct match between the entries in **List-I** to those in **List-II**.

List-I		List-II
(P)		(1) $\frac{5}{4}m\ell^2$
(Q)		(2) $\frac{1}{6}m\ell^2$

Q. 

$\frac{1}{3}mL^2\sin^2\theta$

$I_1 = I_2 = \frac{1}{3}mL^2\sin^2 60 = \frac{1}{4}mL^2$

$I_3 = mr^2 = m(L\cos 30)^2$

$= m\left(L\frac{\sqrt{3}}{2}\right)^2 = \frac{3}{4}mL^2$

$I = \left(\frac{1}{4} + \frac{1}{4} + \frac{3}{4}\right)mL^2 = \frac{5}{4}mL^2$

(A) $P \to 5, Q \to 1, R \to 4, S \to 2$

(B) $P \to 1, Q \to 3, R \to 4, S \to 2$

(C) $P \to 5, Q \to 3, R \to 2, S \to 1$

(D) $P \to 5, Q \to 4, R \to 2, S \to 1$